1. (15 points) True or false:
   
   a. A function which is differentiable at \( x = a \) must also be continuous at \( x = a \).
   \( \boxed{T} \)

   b. The composition of two continuous functions is also continuous.
   \( \boxed{T} \)

   c. Suppose \( f(x) \) is continuous on \([0, 2]\) and \( f(0) = 2, f(2) = 5 \). Then the intermediate value theorem implies \( f(x) \) does not have a root in \((0, 2)\).
   \( \boxed{F} \)

   d. Suppose \( y = a \) is a horizontal asymptote for \( f(x) \). Then the graph of \( y = f(x) \) does not cross the line \( y = a \).
   \( \boxed{F} \)

   e. \( f(x) = \frac{\sin x}{x} \) has a removable discontinuity at \( x = 0 \).
   \( \boxed{T} \)

2. (20 points)
   
   a. Give the \( \varepsilon - \delta \) definition for \( \lim_{x \to a} f(x) = L \).
   
   For any \( \varepsilon > 0 \) there is a \( \delta \) such that
   
   \[ \text{if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon. \]

   b. Use the definition to prove that
   
   \[ \lim_{x \to 3} (2x + 8) = 14. \]
   
   Let \( \varepsilon > 0 \) be given. Choose \( \delta = \varepsilon/2 \). Suppose
   
   \( 0 < |x - 3| < \delta \). We must prove \( |f(x) - L| < \varepsilon \) when
   
   \( f(x) = 2x + 8 \) and \( L = 14 \). But
   
   \[ |f(x) - L| = |2x + 8 - 14| = |2x - 6| = 2|x - 3| < 2\delta = 2\varepsilon. \]

   So \( |f(x) - L| < \varepsilon \) as desired. \( \boxed{\text{II}} \)
3. (20 points) Evaluate the following limits. If the limit does not exist then write DNE.

a. \( \lim_{x \to - \infty} \frac{\sqrt{2x^2+5}}{2x-3} \)  
   \( \text{Since } x < 0 \)  
   \( \chi = -\sqrt{x^2} \)  
   \( \text{Divide top and bottom by } x \)  
   \( \lim_{x \to - \infty} \frac{-\sqrt{1+\frac{5}{x^2}}}{2 - \frac{3}{x}} = -\frac{1}{2} \)

b. \( \lim_{x \to 3^+} \frac{5-3x}{(x-3)(x-5)} \)  
   \( \text{Try } x = 3.001 \)  
   \( x \to \frac{-4}{(0.001)(-2)} \)  
   \( \infty \)

c. \( \lim_{x \to 6} \sin x \)  
   \( \sin 6 \)

d. Suppose \( \lim_{x \to 2} f(x) = 3 \) and \( \lim_{x \to 2} g(x) = -1 \). Evaluate \( \lim_{x \to 2} \frac{f(x)+3g(x)^2}{\sqrt{f(x)}} \)  
   \( \frac{6}{\sqrt{3}} \)
4. (10 points) Neatly sketch the graph of a single function \( f(x) \) which has the following properties:

- \( \lim_{x \to 3^+} f(x) = 2, \lim_{x \to 3^-} f(x) = \infty, \quad f(3) = 1. \)
- \( f(x) \) is continuous from the right at \( x = 5 \) but not continuous from the left at \( x = 5. \)
- \( \lim_{x \to \infty} f(x) = 4, \lim_{x \to -\infty} f(x) = -1. \)
- \( f'(6) = 0. \)
5. (20 points) A ball is tossed and has height in feet given at time $t$ seconds by $y(t) = -t^2 + 6t$.

a. Use the definition of the derivative to prove that $y'(t) = -2t + 6$.

$$y'(t) = \lim_{h \to 0} \frac{y(t+h) - y(t)}{h} = \lim_{h \to 0} \frac{-(t+h)^2 + 6(t+h) - (-t^2 + 6t)}{h}$$

$$= \lim_{h \to 0} \frac{-t^2 - 2th - h^2 + 6t + 6h + t^2 - 6t}{h}$$

$$= \lim_{h \to 0} \frac{-2th + 6h - h^2}{h} = \lim_{h \to 0} -2t + 6 - h$$

$$= -2t + 6$$

b. Find the equation of the tangent line to $y = y(t)$ at the point where $t = 2$.

Point: $(2, 8)$

Slope: $y'(2) = 2$

$Y - 8 = 2(t - 2)$

$$y(t) = 2t$$

$$Y - 8 = 2(t - 2)$$

$c. What is the ball's average velocity from time $t = 1$ to time $t = 3$?

$$\frac{y(3) - y(1)}{3 - 1} = \frac{9 - 5}{2} = 2 \text{ ft/sec}$$

d. How fast is the ball moving when it hits the ground?

$$-t^2 + 6t = 0 \Rightarrow t = 0, 6$$

$$y'(t) = -2t + 6$$

$$y'(6) = -12 + 6 = -6$$

$\text{going left/sec}$

$\text{down}$

e. What is the ball's acceleration?

$$\text{accel} = y'' = -2 \text{ ft/sec}^2$$
6. (15 points) Above is the graph of a function $y = f(x)$.

a. Find the vertical and horizontal asymptotes.

\[
\begin{align*}
\text{V. A.} & : x = 1, \quad x = 3 \\
\text{H. A.} & : y = -2
\end{align*}
\]

b. Evaluate $\lim_{x \to \infty} f(x)$.

$-2$

c. Estimate $f'(0)$.

$2$

d. Estimate $\lim_{x \to \infty} f'(x)$.

$0$

e. On the same axes above carefully sketch a graph of $y = f'(x)$.
Name: