MTH 635 - Differential Geometry - Spring 2016

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Lectures: TR 9:30 am – 10:50 am in Math 122

Office Hours: TR 2:00 pm – 3:30 pm in MATH 111.

Textbook: Riemannian Geometry by Manfredo P. do Carmo

Prerequisite: Undergraduate senior level real analysis, multi-variable calculus, abstract algebra and topology (relevant 400/500 level courses at UB).

Course Description and Material to Be Covered: This is a basic course on Riemannian geometry. We will cover most of Chapters 0-8 and some parts of later chapters of the textbook, supplemented with certain amount of differential topology when necessary. My lectures may not follow exactly the textbook, but I'll try to keep them self-contained, with indexed sections, definitions, theorems, lemmas, etc. So taking class notes is important.

Riemannian geometry is an extension of the Euclidean geometry to Riemannian manifolds, i.e. smooth manifolds equipped with Riemannian metrics. On a Riemainian manifold, one can define the associated intrinsic distance between points and study paths which are locally length minimizing, called geodesics. It is also fundamental to study geometric properties of Reimannian manifolds that are invariant under the isometric equivalence (intrinsic properties), such as volume, curvature, sectional curvatures, and their implications/relations to global topological properties of the underlying manifolds. The basic tool is calculus, i.e. differentiation and integration, but in the first place we need to know how to do these operations on smooth manifolds, i.e. how to extend calculus from Euclidean spaces to smooth manifolds.

I will start from the definition of topological space, then subsequently define or discuss notions (all in C^∞ category) or topics or theorems such as manifolds, real valued functions on manifolds, mappings between manifolds, tangent vectors (with which we can differentiate functions on manifolds), tangent spaces, vector fields, cotangent spaces and 1-forms, general n-forms and their integration on manifolds (briefly), immersion, imbedding and submersion, Riemannian metrics, volume, Levi-Civita covariant derivative (with which we can differentiate vector fields), geodesics, curvature, sectional curvature, Ricci curvature, scalar curvature, Jacobi fields, conjugate points, isometric immersions, Hopf-Rinow theorem, Cartan-Hadamard theorem, spaces of constant curvature, especially hyperbolic geometry.

Grading Scheme: Homework will be assigned along lectures, and collected every three weeks. Your final course grade will be based on the amount of homework you have completed correctly (on your own) and on your class participation.