Second Homework Solutions

10.13. The minimum variance hedge ratio is

$$\beta = \frac{\mathrm{cov}(S_T, F_T)}{\mathrm{var}(F_T)} = \rho\frac{\sigma_G}{\sigma_O} = 0.7\frac{0.20 \cdot 1.50}{0.20 \cdot 1.20} = 0.875,$$

where S_T is the grapefruit juice price and F_T is the orange juice price. Thus the amount to hedge is $0.875 \cdot 150,000 = 131250$ lbs of grapefruit juice. Since Jones is selling, he should short the OJ futures.

$\sigma_{new} = \sqrt{1-\rho^2}\,\sigma_{old} = 0.714\,\sigma_{old}.$

12.2. Show that if an option payoff is given by $\Lambda(S(T)) = S(T)$, then the price of the option is the current stock price S_0. Do this via an arbitrage argument.

If the option price is greater than S_0, buy the stock and write (i.e., sell) an option, so we get money up front. At expiration, sell the stock for $S(T)$ and pay off the option, so the cash flow at time T is zero. We thus have a net gain which violates the no arbitrage principle.

Similarly, if the option price is less than S_0, buy the option and short the stock, so again we gain up front. At time T, exercise the option to get $S(T)$, and use this to buy back the stock, so again the cash flow at time T is zero, and we have an arbitrage. It follows that the option price must be equal to S_0, the stock price.

By 12.1, the price of an option at time 0 is $e^{-rT}\mathrm{E}[\Lambda(S(T))]$. In this problem, we have

$$\begin{aligned} S_0 = e^{-rT}\mathrm{E}[\Lambda(S(T))] &= e^{-rT}\mathrm{E}[S(T))] \\ &= e^{-rT}S_0 e^{\mu T} \qquad \text{by 6.11.} \end{aligned}$$

Thus $S_0 = e^{(\mu-r)T}S_0$. It follows that $e^{(\mu-r)T} = 1$, so $\mu = r$.

14.1. I did most of this in class. $\partial^2 C/\partial\sigma^2$ is negative for small sigma, positive for large sigma, and is 0 at one point, $\hat{\sigma}$. It follows that $\partial C/\partial\sigma$ is increasing for $0 < \sigma < \hat{\sigma}$ and decreasing for $\hat{\sigma} < \sigma < \infty$, so $\partial C/\partial\sigma$ has a unique maximum at $\sigma = \hat{\sigma}$.

14.2. This is just a computation. Compute $\frac{d_1 d_2}{\sigma}$, do some algebra, and end up with $\frac{T-t}{4\sigma^3}(\hat{\sigma}^4 - \sigma^4)$. I don't really feel like typing it up, but will be happy to show you in my office.