Math 459/559 Sample Exam Problems

1. Suppose your utility function is $U(x) = x - 0.04x^2$. You are given two options:
   a) Toss a fair coin. If it comes up heads, you receive \$10, and if it comes up tails you receive nothing.
   b) Take a fixed payment of M dollars.

For which values of M would you take the coin toss? For which would you take the sure payoff?

2. The current price for silver is \$9 per ounce. The storage costs are \$0.24 per year, payable quarterly in advance. Assuming that the interest rate is 10% per year, find the price of a forward contract for silver for delivery in nine months.

3. A company has a \$20 million portfolio with a β of 1.2. Recall that $\beta = \sigma_{PM}/\sigma_M^2$, where M is the market portfolio. The company would like to use S&P 500 futures to hedge. This index is currently at 1080 and each contract is for delivery of \$250 times the index. What is the minimum variance hedge, i.e., how many contracts should be bought or sold?

4. Consider a portfolio of one long call and one short put on the same stock, same K, same expiration, no dividends, and both options European. What are the portfolio's Δ and Γ?

5. Discuss the Monte Carlo method, and how it is used to approximate the price of a European call option.

6. Suppose the spot price of a commodity is \$10, the carrying costs are \$2 per year, paid at the beginning of the year, and that interest is compounded continuously. If the two-year forward price is \$16, find the interest rate.

7. I am going to spend the Spring Semester of 2010 in Norway. I will need to spend about \$15,000. The current exchange rate is \$1=6.81 Norwegian Kroner (NOK). I would like to hedge against exchange rate fluctuations using currency futures, but there is no market in NOK futures. I will hedge using Euros. The current exchange rate is \$1=0.78 Euros. The dollar-euro exchange rate has a standard deviation of 4%, the dollar-NOK exchange rate has a standard deviation of 3.5%, and the correlation coefficient coefficient for the two exchange rates is .85. What position should I take to create a mimimum variance hedge?

8. Consider using the bisector method and Newton's method to approximate $\sqrt{2}$ as a root of $x^2 - 2$. Do three iterations for each method with 1 and 2 as the endpoints in the bisector method and $x_0 = 1$ in Newton's method.

9. Let $C(S,t)$ be the Black-Scholes formula for a call. Evaluate

$$\lim_{\sigma \to 0} C(S,t)$$

and explain why this is the expected result.