Math 459/559 Sample Exam Problems

1. Consider an option whose payoff is

$$\Lambda(S) = \begin{cases} 0 & \text{if } S < K_1 \\ A & \text{if } K_1 \leq S < K_2 \\ B & \text{if } S \geq K_2. \end{cases}$$

Find the price of such an option. Recall that the price of a cash-or-nothing call with payoff $A$ is $Ae^{-r(T-t)}N(d_2)$.

2. Let $P(S,t)$ denote the price of an American put with strike price $K$ and expiration date $T$, and let $S^*(t)$ denote the optimal exercise boundary. Explain why $S^*(T) = K$.

3. Explain why it makes no sense to have $B > K$ for a down-and-out put.

4. For a particular Monte Carlo simulation, an antithetic variables method costs 1.8 times as much per trial as the standard Monte Carlo method. How big a reduction in the variance is required to make the antithetic variables method more efficient?

5. Discuss the effect of dividend payments on a stock's price and on the prices of call and put options. Do not use formulas. Instead describe the direction the price moves compared to a stock which pays no dividend, and give a reason for the movement.

6. Let $P(S,t)$ denote the price of an American put with strike price $K$ and expiration date $T$, and let $S^*(t)$ denote the optimal exercise boundary.

a) Explain why $S^*(T) = K$.

b) If $t_1 < t_2$, show that $S^*(t_1) \leq S^*(t_2)$.

Hints: 1. For a fixed $S$, what happens to $P(S,t)$ as $t$ increases?
2. Explain why $S^*(t) \leq K$ for all $t$.
3. Fix $S = S^*(t_1)$ and look at $P(S,t_2)$.

7. Suppose $f(x) = ax + b$ where $a$ and $b$ are constants. Let $X$ be a standard normal random variable and $Y = -X$ the antithetic variable. Compute the 95% confidence radius for:

a) 20,000 trials using standard a standard Monte Carlo simulation to compute $E[f(X)]$, i.e., no antithetic variable.
b) 10,000 trials using the antithetic variable. This answer may look strange, but why is it not a surprise?