Math 306 - Lab 4
The Glider (aka The Paper Airplane)

This lab is due Thursday, April 16, in class. **Late papers will not be accepted.**

IMPORTANT: The work you submit should be your own and nobody else's. Any exceptions to this will be dealt with harshly.

This lab is based on Lab 5.4 in the text, so the first thing you should do is read page 556. A glider is flying in the xy-plane, $v(t)$ is the glider's speed along its path, and $\theta(t)$ is the angle of inclination of the glider at time t. It is also the angle between the velocity vector and the positive x-axis.

There are three forces acting on the glider: gravity, the lift provided by the wings (perpendicular to the velocity vector), and drag caused by air resistance (parallel to, but in the opposite direction to the velocity vector). Using Newton's second law $F = ma$, one can write down a system of ODE's for d^2x/dt^2 and d^2y/dt^2. After several changes of coordinates, one arrives at the following first-order system for θ and v:

$$\frac{d\theta}{dt} = v - \frac{\cos\theta}{v}$$
$$\frac{dv}{dt} = -\sin\theta - Av^2.$$

Remarkably, this system contains only one parameter, A, which is the "drag-to-lift" ratio, i.e. the drag divided by the lift. Glider designers try to maximize lift and minimize drag, so they try to make A as small as possible. The goal of this lab is to look at solutions to the system and to compare them to the flight of the glider.
Note: You only need to consider positive speeds, i.e. $v > 0$.

1. First consider the case $A = 0$, the glider with zero drag.
   i) Find the equilibria and classify them as (as sinks, centers, saddles, etc.)
   ii) Describe the flight path of the equilibrium solutions in the xy-plane.
   iii) Show that $H(\theta, v) = v^3 - 3v\cos\theta$ is a conserved quantity.
   iv) Use Maple to plot the curves $H =$ constant. See the plot for §5.3 on the Math 306 Maple resources web page for an example of the use of the Maple **contourplot** command.
   v) Use Maple to sketch the solution curves with initial conditions:
      a) $(\theta(0) = 0, v(0) = 0.5)$ b) $(\theta(0) = 0, v(0) = 1.0)$
      c) $(\theta(0) = 0, v(0) = 1.5)$ d) $(\theta(0) = 0, v(0) = 2.0)$
   vi) Describe the behavior of these solutions in the xy-plane.
   vii) Discuss whether there are any qualitative differences in the path of the glider depending on the initial conditions.

2. For $A = 1.0$, Complete parts i), ii), v), vi) and vii) above.

3. Make a paper airplane yourself and test fly it with $\theta(0) = 0$ and different values of $v(0)$. (For some design suggestions, see `http://www.paperairplanes.co.uk`)

a) Is there a qualitative difference in your glider's behavior depending on the initial velocity?

b) Estimate A based on your experiments, and explain how you made your estimate. You may need to make some more Maple plots with different values of A for this part.