The Fourth Annual
Upstate New York Number Theory Conference

SUNY at Buffalo - April 26-27th 2014

Invited Talks

Speaker: Laurent Berger (l'ENS de Lyon, France)
Title: Sen theory and locally analytic vectors
Abstract: I will explain how the theory of locally analytic vectors allows us to generalize Sen theory to extensions whose Galois group is a Lie group of any dimension (joint work with Pierre Colmez).

Speaker: Florian Herzig (University of Toronto, Canada)
Title: On local-global compatibility in the mod p Langlands program.
Abstract: We start by explaining how the known (local) mod p Langlands correspondence for $GL_2(\mathbb{Q}_p)$ occurs in the (global) cohomology of classical modular curves. For $GL_n(\mathbb{Q}_p)$ for $n > 2$ one does not yet have a mod p Langlands correspondence. However, the global setting generalises and one can use it to study expected properties of such a correspondence. In joint work with Stefano Morra we use this global setting to relate two local invariants: one for $GL_3(\mathbb{Q}_p)$-representations and one for mod p Galois representations of the Galois group of $\mathbb{Q}_p$.

Speaker: Xiaoqing Li (SUNY at Buffalo)
Title: Introduction to Eisenstein series.
Abstract: This is an introductory talk to graduate students. In this talk, we will give a survey of the theory of Eisenstein series and its analytic applications, especially to zero free region of L-functions.

Speaker: Tong Liu (Purdue University)
Title: The weight part of Serre's conjecture for $GL(2)$
Abstract: In this talk, I will first review the weight part of Serre's conjecture for $GL(2)$ over totally real field, in particular, the situation over $\mathbb{Q}$.

Then I will roughly explain how to reduce to the conjecture to a purely local question: To study the reduction of certain two-dimensional crystalline representations (pseudo-Barsotti-Tate representations), which eventually lead the proof of the conjecture under the usual Taylor-Wiles hypothesis. This is a joint work with Toby Gee and David Savitt.

Speaker: Alice Medvedev (CUNY)
Title: TBA
Abstract: TBA

Speaker: Steven Sperber (University of Minnesota)
Title: Arithmetic Properties of A-Hypergeometric Systems
Abstract: Classical nonconfluent hypergeometric differential equations arise often as the Picard-Fuchs differential equations associated with families of algebraic varieties. As such there is often an interesting interplay between arithmetic properties of the families viewed say over a finite field of characteristic p and analytic properties of these equations and their solutions. Confluent cases arise as well in the study of families of exponential sums defined over finite fields. A-hypergeometric or GKZ systems of parial differential equations at once generalize classical hypergeometric equaions and also provide new links to arithmetic in these cases. In joint works with Adolphson, we prove general formulas for the unique unit root of a toric exponential sum. We also connect quite generally Hasse invariants with mod p solutions of Picard-Fuchs equations.

Speaker: Dinesh Thakur (University of Rochester)
Title: Zeta and Multizeta values in function field arithmetic
Abstract: I will try to give a perspective and an overview of results, conjectures and questions about this topic.

Contributed Talks

Speaker: John Bergdall (Boston University)
Title: Ordinary representations on $U(3)$ and a conjecture of Breuil and Herzig
Abstract: Breuil and Herzig have conjecturally described a certain unitary representation appearing inside a space of p-adic modular forms which is meant to describe a piece of local-global compatibility in the case of p-ordinary automorphic forms for unitary groups. In this talk we will describe recent progress in proving their conjectures in the case $U(3)$. This is joint

work with Przemyslaw Chojecki.

Speaker: Peter Cho (SUNY at Buffalo)
Title: n-level densities of Artin L-functions
Abstract: We consider a family of twisted Artin L-functions, $L(s, \pi \times \rho)$, where π is a fixed self-dual cuspidal representation of GL_m, and ρ is given by $L(s, \rho, K) = \frac{\zeta_K(s)}{\zeta(s)}$ attached to an S_{d+1}-field K. By the strong Artin conjecture, we consider ρ as a cuspidal representation of GL_d. We obtain n-level densities for our families under certain counting conjectures. Our result is unconditional for S_3-fields regardless of π, which is of symplectic type or of orthogonal type. For π of orthogonal type (i.e., the symmetric square L-function has a pole at $s = 1$), the n-level density computation is unconditional for S_4-fields (and S_5-fields under the strong Artin conjecture for ρ.). This is a joint work with Henry Kim.

Speaker: Douglas Haessig (University of Rochester)
Title: Dwork's unit root L-function in the rank one case
Abstract: We will discuss some work-in-progress concerning unit root L-functions, specifically, ones coming from families of exponential sums.

Speaker: Jamie Juul (University of Rochester)
Title: Periodic points and iterated Galois groups
Abstract: The Chebotarev Density Theorem allows us to relate questions about the proportion of periodic points of a polynomial map over Z/pZ to questions about fixed points of iterated Galois groups. I will discuss this relationship and describe conditions under which the proportion of periodic points must approach 0 as p becomes large. This represents joint work with Kalyani Madhu and Tom Tucker.

Speaker: Jinghao Li (Binghamton University)
Title: Purity Results on F-crystals
Abstract: We will present a survey of purity results for stratifications in positive characteristic associated to F-crystals, including some recent new purity results.

Speaker: Daniel Miller (Cornell)
Title: Average ranks of Selmer groups and maximal isotropic subspaces
Abstract: Last year Bhargava and Shankar gave an upper bound for the average rank of an elliptic curve by bounding the average rank of their 2-Selmer group. In fact, there is a very precise conjecture on the distribution of the cardinality of Selmer groups of elliptic curves, due to Poonen and

Rains, that is motivated by realizing the Selmer group as an intersection of infinite-dimensional maximal isotropic subspaces. I will review Poonen and Rains' motivation, give a precise formulation of their conjecture, and discuss some of its consequences.

Speaker: Brendan Murphy (University of Rochester)
Title: Generalized Kloosterman Sums and the Structure of Finite Rings
Abstract: A Kloosterman sum is an exponential sum whose estimates are important in many problems in number theory, for instance in estimating the Fourier coefficients of modular forms. The most famous such estimate is Weil's bound, which says that the magnitude of a Kloosterman sum over Z/p is roughly the square root of number of terms in the sum, where p is prime. For composite n, Kloosterman sums over Z/n do not enjoy square root cancellation, so one might ask if square root cancellation characterizes fields. In this talk I will explain my recent work, joint with Alex Iosevich and Jonathan Pakianathan, where we study Kloosterman sums over (possibly non-commutative) finite rings and show that asymptotically, rings with square root cancellation must be fields.

Speaker: Christopher Rasmussen (Wesleyan University)
Title: Picard curves with good reduction away from 3
Abstract: Curves with good reduction away from one prime are often candidates for unusual arithmetic and geometric behavior, but they are also hard to come by in practice, outside of a few well-known families. We present joint work with Beth Malmskog, determining all Picard curves with good reduction away from 3, by generalizing an approach used by Nigel Smart in the 1990's to find genus 2 curves with good reduction outside 2.

Speaker: James Ricci (Wesleyan University)
Title: Regular Quadratic Polynomials of Fixed Conductor
Abstract: In 1924, Helmut Hasse established a local-to-global principle for representations of rational quadratic forms. Unfortunately, an analogous local-to-global principle does not hold for representations over the integers. A quadratic polynomial is called regular if such a principle exists; that is, if it represents all the integers which are represented locally by the polynomial itself over $\mathbb{Z}_p$ for all primes p as well as over $\mathbb{R}$. In 1953/54, G.L. Watson showed that up to equivalence, there are only finitely many primitive positive definite integral regular quadratic forms in three variables. More recently, W.K. Chan and B.-K. Oh take the first step in understanding regular ternary quadratic polynomials by showing that there are only finitely many primitive positive regular triangular forms in three variables. In this talk, I will

give an analogous finiteness result for regular ternary quadratic polynomials in greater generality. By defining an invariant called the conductor and a notion of a semi-equivalence class of a quadratic polynomial, we will utilize the theory of quadratic forms to obtain the following result: Given a fixed conductor, there are only finitely many semi-equivalence classes of positive regular quadratic polynomials in three variables.

Speaker: Adam Towsley (CUNY)
Title: Endomorphisms of Bounded Height and Resultant
Abstract: Let K be an algebraic number field and $B \geq 1$. For an Endomorphism $\varphi : \mathbb{P}^n \to \mathbb{P}^n$ defined over K of degree d let $\mathfrak{R}_\varphi$ denote its minimal resultant ideal. For a fixed height function $h_{\mathcal{M}_d^n}$ on the moduli space of dynamical systems this paper shows that all such morphisms φ of bounded resultant $\mathrm{N}_{K/\mathbb{Q}}(\mathfrak{R}_\varphi) \leq B$ and bounded height $h_{\mathcal{M}_d^n}(\langle\varphi\rangle) \leq B$ are contained in finitely many $\mathrm{PGL}_{n+1}(K)$-equivalence classes. This answers a question of Silverman in the affirmative.

Speaker: John Voight (Dartmouth)
Title: The group $PSU(3,5)$ as a Galois group
Abstract: We realize explicitly the group $PSU(3,5)$ as regularly as a Galois group by numerical computation of a degree 50 three-point branched cover of the projective line. We will discuss this computation and other remarkable properties of the rational map.

Speaker: Xiao Xiao (Utica College)
Title: Subtle Invariants of F-crystals
Abstract: The isomorphism number n_D of a p-divisible group D over an algebraically closed field of characteristic $p > 0$ is the smallest non-negative integer n such that if the n-th truncation of another p-divisible group C (whose dimension and codimension are the same as D) is isomorphic to the n-th truncation of D, then C is isomorphic to D. The isomorphism number $n_{\mathcal{M}}$ of an F-crystal $\mathcal{M}$ is the generalization of n_D to F-crystals. In order to estimate $n_{\mathcal{M}}$, Vasiu introduced a computable invariant called level torsion $l_{\mathcal{M}}$ of $\mathcal{M}$, and proved that $n_{\mathcal{M}} \leq l_{\mathcal{M}}$ and further conjectured that $n_{\mathcal{M}} = l_{\mathcal{M}}$. Recent work of Gabber, Lau, Nicole and Vasiu proved that $n_D = l_D$ for all p-divisible groups D and a very sharp (in some cases, optimal) upper bound of n_D has been discovered. Motivated by their work, I will explain why Vasius conjecture $n_{\mathcal{M}} = l_{\mathcal{M}}$ should also be true in this talk. This will open up many doors of finding optimal upper bounds of $n_{\mathcal{M}}$ in general.

Speaker: David Zywina (Cornell)

Title: Elliptic surfaces and the Inverse Galois Problem.

Abstract: By studying the Galois action on étale cohomology groups arising from elliptic surfaces, we will prove several new cases of the Inverse Galois Problem. In particular, we will explain why each of the simple groups $\mathrm{PSp}_4(\mathbb{F}_p)$ occur as the Galois group of some Galois extension of $\mathbb{Q}$. The key ingredients will be a big monodromy result along with some known cases of the Birch and Swinnerton-Dyer conjecture.