Math 458/558 Sample Second Exam Problems

1. Consider an additive model $S(k+1) = aS(k) + u(k)$, where $a$ is a positive constant and the $u(k)$ are independant normal random variables with mean 0 and variance $\sigma^2$. Assume $S(0)$ is known, and find $E[S(5)]$ and the variance of $S(5)$.

2. Suppose that $X(t)$ follows the process $dX = 3X\,dt + 4X\,dz$. Using Ito's Lemma, find the equation for the process for $Y(t) = X(t)^n$.

3. Suppose the price for an option is given by $f(S,t) = 10e^{-r(T-t)}N(d_2)$, where

$$d_2 = \frac{\ln\left(\frac{S}{40}\right) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}.$$

Find the value at expiration, i.e. evaluate $\lim_{t\to T-} f(S,t)$ as a function of $S$.

4. Create a two-month binomial lattice model to find the value of a **European** put option with strike price $K = 55$, intial stock price $S(0) = 54$, volitility $\sigma = 0.33$, $\nu = 0.13$, and annual interest rate $r = 0.12$. Use one month for $\Delta t$.

5. a) Define $\Delta$, $\Gamma$ and $\Theta$ for a derivative whose price is given by $f(S,t)$.

b) Explain why one wants to have $\Delta = 0$ for an options portfolio, and how $\Gamma$ is related to this goal.

6. Suppose that $S(t)$ follows the process $dS = \alpha S\,dt + \beta S\,dz$, where $\alpha$ and $\beta$ are constants. Find the equation of the process for $Y(t) = e^{r(T-t)}/S$.

7. Create a two-month binomial lattice model to find the value of a **American** put option with strike price $K = 55$, intial stock price $S(0) = 54$, volitility $\sigma = 0.33$, $\nu = 0.13$, and annual interest rate $r = 0.12$. Use one month for $\Delta t$.

8. Let $C$ denote the price of a call option with strike price $K$ on a stock with price $S$. Let $r > 0$ denote the risk-free interest rate, and assume continuous compounding.

a) Make a portfolio which consists of buying the call, shorting the stock, and investing $K$ in the risk-free asset. Use this portfolio and an arbitrage argument to show that $C > S - K$.

b) What does this say about the relation between American and European call options?

9. Show that for any consant $a$, there is a constant $C_a$ such that $e^{C_a t}S^a$ is a solution to the Black-Scholes equation.

10. A **strap** is a spread created by buying a European put and two European calls, all with the same strike price and expiration date.

a) Sketch the payoff as a function of $S$.

b) When purchasing a strap, what view is the investor taking?

11. Suppose a stock is currently selling for \$25. A one-year European call with strike price \$24 costs \$6, and a one-year European put with strike price \$24 costs \$2. If interest is compounded continuously, find the interest rate $r$.