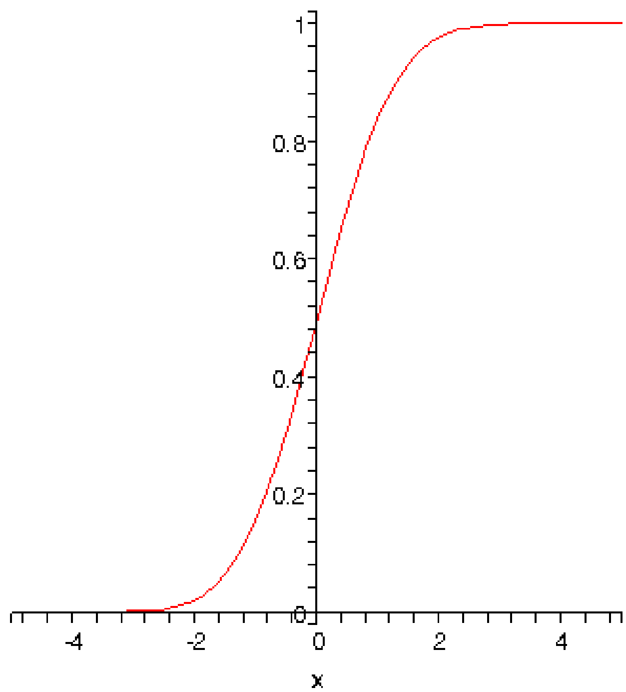
```
> # exact solution of B-S for price of Euro (also American) call option
  CSt_formula := S*N(d[1]) - K*exp(-r*(T-t))*N(d[2]);
```

$$CSt\_formula := S\,N(d_1) - K\,\mathbf{e}^{(-r\,(T-t))}\,N(d_2)$$

```
> N_function := x->(1/2)*(1+erf(x/sqrt(2)));
  plot(N_function(x),x=-5..5);
```

$$N\_function := x \rightarrow \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)$$




```
> d1_function := t -> (log(S/K) + (r + (1/2)*sigma^2)*(T-t))
  /(sigma*sqrt(T-t));
  d2_function := t -> (log(S/K) + (r - (1/2)*sigma^2)*(T-t))
  /(sigma*sqrt(T-t));
```

$$d1\_function := t \rightarrow \frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d2\_function := t \rightarrow \frac{\log\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

```
> CSt := subs(d[1]=d1_function(t),d[2]=d2_function(t),CSt_formula);
  CSt := eval(subs(N=N_function,%));
```

$$CSt := S\,N\left(\frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}\right) - K\,\mathbf{e}^{(-r(T - t))}\,N\left(\frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}\right)$$

$$CSt := S\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\left(\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T - t)\right)\sqrt{2}}{2\,\sigma\sqrt{T - t}}\right)\right) - K\,\mathbf{e}^{(-r(T - t))}\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\left(\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T - t)\right.}{2\,\sigma\sqrt{T - t}}\right.\right.$$

```
> # Exercise L10: 6 month call, r = 0.04
  # L.10 Use a binomial lattice to find the value of a value of a six-month
  call option with strike price # # $52, Current price $55, interest rate 4%,
  standard deviation 0.25, and Delta t= 1 month.
  # L.11 Use a binomial lattice to find the value of a value of a six-month
  American put and a six-month #  #  European put option with the same
  parameters as in exercise L. 10. Fri 11/13
  #  Read: Sec 13.1, 13.2, 13.3, 13.5.
  #  Homework: 13.2, 13.4
  # L.12 Use the Black-Scholes formulas to compute the values of a
  European call option and European put # #  option with the same
  parameters as in Exercise L.10. (Show your work.)
  T_value := evalf(6/12);
  sigma_value := 0.25;
  r_value := 0.04;
  params := {K = 52, sigma = sigma_value, r = r_value, T = T_value};
```

$$T\_value := 0.5000000000$$

$$sigma\_value := 0.25$$

$$r\_value := 0.04$$

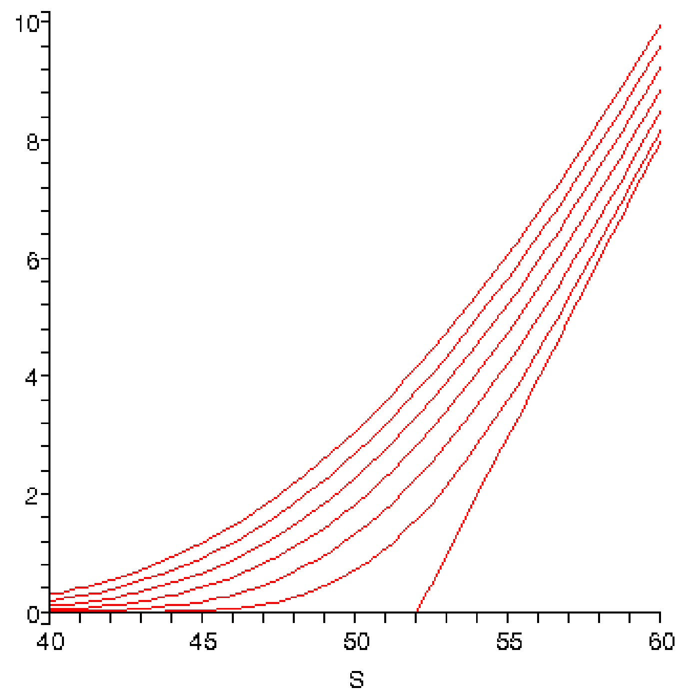
$$params := \{K = 52, \sigma = 0.25, r = 0.04, T = 0.5000000000\}$$

```
> CSt_params := subs(params,N=N_function,CSt);
```

$$CSt\_params := S\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{2.000000000\left(\ln\left(\frac{1}{52}S\right) + 0.03562500000 - 0.07125000000\,t\right)\sqrt{2}}{\sqrt{0.5000000000 - t}}\right)\right)$$

$$- 52\, e^{(-0.02000000000 + 0.04\, t)} \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{2.000000000 \left(\ln\left(\frac{1}{52} S \right) + 0.004375000000 - 0.00875000000\, t \right) \sqrt{2}}{\sqrt{0.5000000000 - t}} \right) \right)$$

```
> plot0 := plot(subs(t=0,CSt_params),S=40..60):
  plot1 := plot(subs(t=1/12,CSt_params),S=40..60):
  plot2 := plot(subs(t=2/12,CSt_params),S=40..60):
  plot3 := plot(subs(t=3/12,CSt_params),S=40..60):
  plot4 := plot(subs(t=4/12,CSt_params),S=40..60):
  plot5 := plot(subs(t=5/12,CSt_params),S=40..60):
  plot6 := plot(subs(t=6/12,CSt_params),S=40..60):
  with(plots):
  display([plot0,plot1,plot2,plot3,plot4,plot5,plot6]);
  subs(S=55,t=0,CSt_params);
  evalf(%);
```



$$\frac{55}{2} + \frac{55}{2} \operatorname{erf}\left(2.828427124 \left(\ln\left(\frac{55}{52} \right) + 0.03562500000 \right) \sqrt{2} \right)$$
$$- 52\, e^{(-0.02000000000)} \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(2.828427124 \left(\ln\left(\frac{55}{52} \right) + 0.004375000000 \right) \sqrt{2} \right) \right)$$

$$6.086049735$$

```
> # Check by substitution that CSt(S,t) actually satisfies Black-Sholes
  equation
  BS_pde := diff(f(S,t),t)+r*S*diff(f(S,t),S) +
  (1/2)*sigma^2*S^2*diff(diff(f(S,t),S),S) - r*f(S,t) = 0;
  print(` If f(S,t) = CSt is a solution the following should give 0 = 0`);
  simplify(subs(f(S,t)=CSt,BS_pde));
```

$$BS\_pde := \left(\frac{\partial}{\partial t} f(S,t)\right) + r\,S\left(\frac{\partial}{\partial S} f(S,t)\right) + \frac{1}{2}\sigma^2 S^2\left(\frac{\partial^2}{\partial S^2} f(S,t)\right) - r\,f(S,t) = 0$$

If f(S,t) = CSt is a solution the following should give 0 = 0

$$0 = 0$$

```
> # Use put-call parity to determine price of Euro. put
  PSt := CSt + K*exp(-r*(T-t)) - S;
```

$$PSt := S\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\left(\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)\right)\sqrt{2}}{2\,\sigma\sqrt{T-t}}\right)\right) - K\,\mathbf{e}^{(-r(T-t))}\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\left(\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)\right)}{2\,\sigma\sqrt{T-t}}\right)\right) + K\,\mathbf{e}^{(-r(T-t))} - S$$

```
> print(` Uf PSt is a solution the following sould give 0 = 0`);
  simplify(subs(f(S,t)=PSt,BS_pde));
```

Uf PSt is a solution the following sould give 0 = 0

$$0 = 0$$

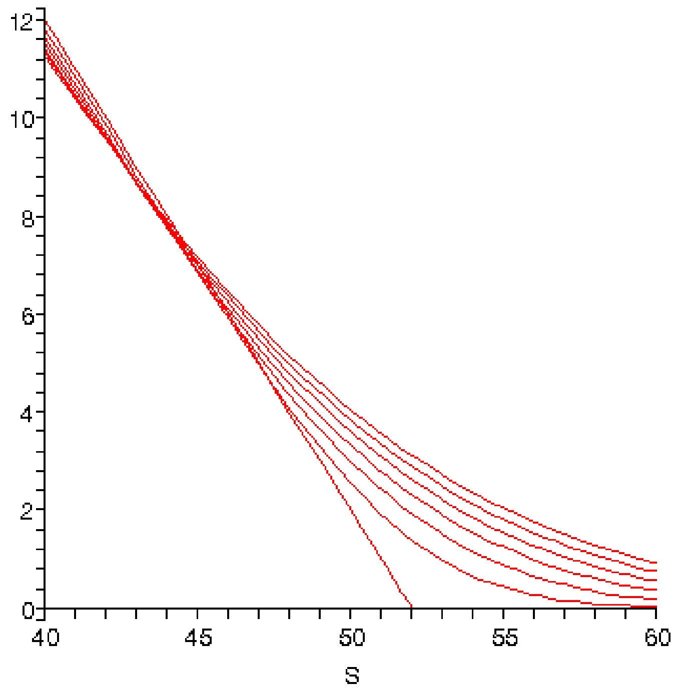
```
> PSt_params := subs(params,PSt);
```

$$PSt\_params := S\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{2.000000000\left(\ln\left(\frac{1}{52}S\right) + 0.03562500000 - 0.07125000000\,t\right)\sqrt{2}}{\sqrt{0.5000000000 - t}}\right)\right) - 52\,\mathbf{e}^{(-0.02000000000 + 0.04\,t)}\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{2.000000000\left(\ln\left(\frac{1}{52}S\right) + 0.004375000000 - 0.00875000000\,t\right)\sqrt{2}}{\sqrt{0.5000000000 - t}}\right)\right) + 52\,\mathbf{e}^{(-0.02000000000 + 0.04\,t)} - S$$

```
> plot0 := plot(subs(t=0,PSt_params),S=40..60):
  plot1 := plot(subs(t=1/12,PSt_params),S=40..60):
  plot2 := plot(subs(t=2/12,PSt_params),S=40..60):
```

```
plot3 := plot(subs(t=3/12,PSt_params),S=40..60):
plot4 := plot(subs(t=4/12,PSt_params),S=40..60):
plot5 := plot(subs(t=5/12,PSt_params),S=40..60):
plot6 := plot(subs(t=6/12,PSt_params),S=40..60):
with(plots):
display([plot0,plot1,plot2,plot3,plot4,plot5,plot6]);
subs(S=55,t=0,PSt_params);
evalf(%);
```

Error, numeric exception: division by zero



$$-\frac{55}{2}+\frac{55}{2}\operatorname{erf}\left(2.828427124\left(\ln\left(\frac{55}{52}\right)+0.03562500000\right)\sqrt{2}\right)$$
$$-52\,e^{(-0.02000000000)}\left(\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(2.828427124\left(\ln\left(\frac{55}{52}\right)+0.004375000000\right)\sqrt{2}\right)\right)+52\,e^{(-0.02000000000)}$$

$$2.056380745$$

>