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Why the count de Borda cannot beat the Marquis de Condorcet

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Abstract Although championed by the Marquis the Condorcet and many others, majority rule has often been rejected as indeterminate, incoherent, or implausible. Majority rule's arch competitor is the Borda count, proposed by the Count de Borda, and there has long been a dispute between the two approaches. In several publications, Donald Saari has recently presented what is arguably the most vigorous and systematic defense of Borda ever developed, a project Saari has supplemented with equally vigorous objections to majority rule. In this article I argue that both Saari's objections to majority rule and his positive case for the Borda count fail. I hold the view that defenders of Condorcet cannot muster arguments to convince supporters of Borda, and vice versa, but here I am only concerned to show that the Count de Borda cannot beat the Marquis de Condorcet. Saari's approach displays what I take to be widespread fallacies in reasoning about social choice worthy of closer analysis. This debate bears on important questions in the philosophy of social choice theory.

1 Introduction

1.1 Majority rule is simple if groups choose between two alternatives. The only complication is that there may be a tie. Matters are more complicated if groups choose among more than two alternatives. Suppose Tom, Dick, and Harry rank A, B, and C as follows: Tom ranks them (A, B, C), Dick (C, A, B), and Harry (B, C, A). (I write "(A, B, ...)" for rankings, and "{A, B, ...}" for sets. By "rankings," I mean "ordinal rankings," rankings that do not convey any information about alternatives other than to identify alternatives to which they are preferred—cf. [17] and [8], chapter 1 for more precise treatment.) Suppose they opt to determine a ranking by taking pairwise majority votes. Yet since A beats B, B beats C, and C beats A, no

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ranking emerges; instead, we obtain a cycle. This is the Condorcet paradox. Majority rule as sketched is *indeterminate*: it does not always deliver a result. Arrow's Impossibility Theorem, in one way of thinking about it, generalizes this phenomenon, isolating those features of majority rule that imply that we sometimes need to ascribe preferences to the group that do not constitute a ranking. In light of these results, some (such as [4] and [18]) have argued that majoritarian democracy is conceptually flawed, insisting that we do not have a coherent majoritarian way of making decisions if there are more than two alternatives.

Either because of these troubles or because they find it independently more plausible, some support majority rule's arch-competitor, the *Borda count.* Suppose a group must rank m candidates, given that each individual has already ranked them. Borda has each individual assign 0 to her last-ranked candidate, 1 to the second-to-the-last-ranked, until she assigns $n-1$ to her top-ranked, and then, for each of the candidates, sums over those numbers to determine the group ranking. The question is: should we abandon majoritarian decision making as incoherent or otherwise implausible and adopt the Borda Count instead?

Donald Saari, for one, thinks we should, and has defended this view forcefully in recent publications (such as [10, 11, 12, 15, 16]). Saari is one of the most distinguished mathematical contributors to voting theory, and his is conceivably the most vigorous and sophisticated defense of Borda ever undertaken. Much is at stake. If there is an overwhelming case for Borda, then, whenever we are using another method for aggregating preferences, we make some people "losers" though they would have been "winners" had the most defensible method been used, and *mutatis mutandis* for majority rule. At the same time, if there is no case championing one method over others, there will always be people who are "losers" though they would have been "winners" had another, equally reasonable method been used. This debate is old: the Marquis de Condorcet and the Count de Borda, French noblemen in troubled times, debated these questions already in the late 18th century, a golden age of reflection on group decision making. As far as legitimacy of collective decisions is concerned, Saari's defense of Borda, if successful, would constitute a great insight.

1.2 However, Saari's defense fails, and my goal is to show why. In earlier work ([7]) I have defended a conception of majoritarian decision making that demonstrates that, contrary to the kind of criticism mentioned in the very first paragraph, there is indeed a coherent majoritarian decision rule that solves the indeterminacy problem. Since my proposal (which *qua* aggregation mechanism had already been discussed elsewhere in the literature) bears affinities to ideas of Condorcet, I call it the *Condorcet Proposal* (and here refer to it simply as "the Proposal"). I have also argued there for the *multiplicity thesis*, the claim that in the same situation different methods may be reasonable. I hold this view especially for preference aggregation: it is indeed the case that in such scenarios, some people will be "losers" although they would not have been had another, equally reasonable decision rule been adopted. So while majoritarian decision making is conceptually sound, it *never* is the uniquely reasonable method. Saari, as far as I can tell, does not think that majoritarian decision making as captured by the Proposal is incoherent. He does, however, think that it has several features that render it deeply implausible, and that, at any rate, the Borda Count can be derived from ideas so basic and compelling that even prima-facie supporters of the Proposal have to endorse them.

My goal is to refute that view. The Count cannot beat the Marquis: defenders of the Borda Count have no arguments that should make supporters of the Proposal change their minds. The Marquis cannot beat the Count either, but that aspect of the debate is not my concern here. Even the first direction ("the Count cannot beat the Marquis") I cannot fully establish here since I do not claim for this paper to have explored *all* arguments given on behalf of Borda. But while I do make this larger claim as well, my concern here is with Saari's stimulating and prominent defense of Borda. Saari's reasoning displays widespread fallacies, not mathematical fallacies, but fallacies in the philosophical reasoning *about* mathematical insights. So what is conceivably the most promising attempt ever to show that Borda is the uniquely reasonable aggregation method when ordinal rankings are to be aggregated, *fails.* Yet there is more to this. Saari's arguments fail because they reflect an inadequate view of what counts as a successful argument in reasoning about group decision rules, a theme that this article will spell out in some detail. Reflection on such matters, and hence this part of the philosophy of social choice theory, I believe, deserves much more investigation than has so far been conducted.

1.3 Section 2 discusses both aggregation methods. Section 3 discusses Saari's objections and shows that they fail. Section 4, the argumentatively central section, analyzes Saari's case for Borda and shows that his arguments will not convince supporters of the Proposal. In particular, Section 4 argues that an (otherwise highly illuminating) argument against basing aggregation merely on information contained in pairs that Saari has presented in several publication *fails to have any bearing whatsoever* on this debate. His arguments speak to majority rule as introduced in 1.1 but not to the Proposal, and we have long known that the coherence of majoritarian decision making cannot be preserved if we conceive of it in terms of that rule. My main reference is [14]. While Saari develops many of his arguments elsewhere as well, that paper contains a particularly useful summary of his objections to the Proposal and his arguments in support of Borda. Unless otherwise noted, references to Saari are to that article.

A quick note on terminology: To have different names, I call the rule introduced in the very first paragraph "majority rule," but argue that the Condorcet Proposal is what we should mean by majoritarian decision making. While "majority rule" is disqualified by the indeterminacy problem, we do have a coherent account of majoritarian decision making. I cannot emphasize enough the importance of keeping apart majority rule and the Condorcet Proposal, since different arguments will often apply to them. Also, the reader should keep in mind that, since I announce Condorcet a contender whereas Saari pronounces Borda a champion, our disagreement does not fully re-instantiate "Condorcet vs. Borda." The disagreement, rather, is about the multiplicity thesis.

2 Condorcet and Borda

2.1 Let me begin by introducing the Proposal. Suppose we must rank m alternatives in a majoritarian manner. The Proposal looks at all $m(m-1)/2$ pairs among the alternatives and selects one or more of their $m!$ rankings in light of these pairwise votes, regardless of cycles. Those votes being the "data," we ask which ranking they support best. The Proposal selects rankings *supported by a maximal number of votes in pairwise votes*. For each ranking R, we look at the $m(m-1)/2$

pairs and count the voters ranking the respective alternatives in the same way as R and thus *supportR*. Suppose a group of 48 must rank A, B, and C. Ten people rank them (A, B, C), 12 (A, C, B), 5 (B, A, C), 7 (B, C, A), 3 (C, B, A) and 11 (C, A, B). So we have 3!=6 rankings and 3(3−1)/2=3 pairwise votes. The ranking with highest support is (A, C, B): In A vs. B, 33 people support it (33 people rank A over B), in B vs. C 26 people, and in A vs. C 27. So 86 votes support (A, C, B), compared to 82 for (A, B, C), 64 for (B, A, C), 58 for (B, C, A), 62 for (C, B, A), and 80 for (C, A, B).

The Proposal differs from majority rule as introduced in 1.1: that rule concatenates results of pairwise votes, which sometimes leads to a collective ranking and sometimes not. As opposed to that, the Proposal by construction always selects a ranking as collective outcome. In fact, the Proposal selects a collective ranking regardless of whether voters themselves are assumed to have rankings. This feature gains center stage in Section 4. To wit: Saari has developed what he takes to be a crucial objection to the Proposal that draws on this feature, whereas I will argue in Section 4 that that objection is a non-starter.

One may make three claims about the Proposal: first, that it captures what, in ideal theory, we should mean by *majoritarian* decision making; second, that it is a *reasonable* method for aggregating rankings; and third, that it is the *most or uniquely reasonable* such method. I defend the first and second, but reject the third claim. A reader willing to grant the first and the second point, or mostly concerned with Saari's objections to majoritarian decision making and my objections to his account, should proceed to Section 3 (and perhaps briefly stop at the beginning of 2.2 for a more careful introduction of the Borda Count). A reader interested in the issues involved in actually establishing those claims and in the multiplicity thesis (for which the third claim is relevant) should read this section as well. However, since I have argued for these views elsewhere ([7]), I will only sketch some important bits of the argument.

Let me begin by arguing for the first claim. Some may not think that this claim is much in need of argumentative support. Yet for one thing, majority rule straightforwardly merely applies to two alternatives, and any claim that some proposal captures what we should mean by majoritarian decision making will have to be argued for. Also, if this claim is right, it entails that Arrow's theorem does not address majoritarian decision making at all. For the Proposal violates Arrow's Independence condition, which *therefore* (in virtue of my first claim) does not hold for majoritarian decision making. It also follows that majoritarian decision making is *coherent*.

[7] offers two lines of argument for the claim that the Proposal is a distinct and distinguished majoritarian rule with. First, I argue that the Proposal has formal features rendering it such a proposal. Second, I observe that common arguments for majority rule only address cases of two alternatives. I show that several such arguments generalize to support the Proposal. This is a conditional claim: While I argue that standard argument for majority rule generalize and then support the proposal, I do not *thereby* endorse those arguments. What unifies both of these lines of argument is the idea to show that the Proposal possesses features one would intuitively think a general majoritarian rule should have, given what we know about majority rule for the case of two alternatives. It is useful to have two parts of the second line in place. One argument for majority rule (for two alternatives) is

> **Maximization:** Majority rule maximizes the number of people who exercise self-determination. This argument evidently generalizes to whichever property one thinks is expressed in the act of voting or realized by winning an election.

Strictly speaking, Maximization does not generalize. Self-determination is realized in votes, but rankings are not subject to voting, according to the Proposal. Yet the Proposal maximizes the *number of voting acts* expressing self-determination, rather than the *number of people* who express it. For two alternatives, this argument is the original Maximization. The generalization should be convincing to whomever Maximization was convincing. (I hasten to add that I will argue below that the Borda Count is also supported by a 'maximization' claim. Like Saari (cf. [14], p 337), I think that claims to maximization are standard, in the sense that any reasonable decision procedure will maximize something of interest. I trust, however, that this sketch makes clear what role I think Maximization plays in the overall argument in support of the claim that the Condorcet Proposal is what we should mean by majoritarian decision making.) Another argument is

> **Condorcet's Jury Theorem:** Supposes it makes sense to speak of being right or wrong about political decisions. Suppose n agents choose between two alternatives; that each has a probability of $p>1/2$ of being right; and that their probabilities are independent of each other (i.e., they make up their minds for themselves). Then, as n grows, the probability of a majority's being right approaches 1.

The theorem only applies to two alternatives. However, as [19] and [20] show, there is a generalization, which can draw on ideas of Condorcet himself, picking out precisely the rankings with maximal support. My conclusion, again, is that the Proposal captures what we should mean by majoritarian decision making.

The second claim is that the Proposal is a reasonable rule for aggregating rankings. Our strategy is no longer to identify features that, intuitively, a general *majoritarian* rule should have, but to identify features that, intuitively, a *reasonable* rule should have. If one does not press too much on what makes for "reasonableness" in abstraction from investigating proposed features of voting rules, there is an obvious way of assessing whether majority rule is such a rule, namely by taking arguments for majoritarian decision making (and their generalizations) at face value (which was not required for claiming that the Proposal *constitutes* majoritarian decision making). This takes us a long way, but rather than exploring this route, I make a different point. Recall that the same rankings emerge when we apply the Proposal or the generalized Jury Theorem. There is a third approach identifying those rankings. That approach (due to [6] and also known as Kemeny's rule) searches for a *compromise* among rankings. Forming their "average" suggests itself. This operation presupposes a notion of distance between any two rankings. Define this distance as the number of pairs with regard to whose ranking they differ. The distance between (A, B, C) and (B, A, C) is 1: they differ only with regard to $\{A, B\}$. A suitable conceptualization for an average of rankings is their *median* relative to this metric, the ranking minimizing the sum over the distances from the rankings. This median is also the result of the maximum likelihood method and the recommendation of the Proposal. Strikingly, three very different methods select the same rankings. Especially the fact that the rankings selected by the Proposal emerge through an intuitive notion of compromise supports the claim that it is a reasonable method. (Some theorists might say that the Condorcet Proposal is "really" Kemeny's rule in disguise: but that is true only if one takes approaches to group decision making as identical if they always deliver the same outcomes, regardless of the procedure they apply to that end.)

2.2 Before explaining why I reject the third claim ("the Proposal is the uniquely reasonable rule"), I say some more about the *Borda Count* introduced in 1.1. For an example, suppose three people rank four alternatives as follows: person 1 (*A*, *B*, *C*, *D*), 2 (*B*, *C*, *D*, *A*), and 3 (*A*, *B*, *D*, *C*). The social ranking, according to Borda, is then (*B*, *A*, *C*, *D*): *A* obtains six points, *B* seven, *C* three and *D* two. For an equivalent description of Borda, suppose all votes between any two alternatives are taken. Then for each alternative, we count the number of elections in which any agent prefers this alternative to the respective competitor. If A obtains a count of 23, then in 23 cases some voter, confronted with a pair including *A*, prefers *A*. A third characterization of Borda is that it ranks alternatives starting with the one with the highest *average position* across rankings. Ranking alternatives by their average position across rankings, Borda asks about the *support for each of the alternatives across rankings*, whereas Condorcet asks about the *support for each of the rankings across pairwise elections.*

Saari claims Borda is the preferred rule for aggregating rankings. I claim that, whenever we aggregate rankings, Condorcet would not convince Borda, and vice versa. I recapitulate my argument only with regard to the Jury Theorem and Maximization, and only present that bit showing that Borda remains unconvinced by arguments for the Proposal. This argument (once completed) shows (a) that the third claim about the Proposal is false (it is not the uniquely reasonable rule), and (b) that a symmetric version of the second claim is true for Borda (it is a reasonable rule as well), whereas (c) a parallel version of the third claim for Borda is also false (Borda is not the uniquely best method). According to the Jury Theorem, rankings selected by the Proposal bestow the highest likelihood on the election result. Yet the alternative ranked highest by Borda is the *single alternative* that, if best, bestows highest probability upon voting results, or at least this is so if the voters' competence *p* can be assumed to be close to 1/2 (cf. [19] and [20].) Recall that the Proposal selects *rankings* with highest support, and Borda ranks *alternatives* in terms of their support. So Borda aims to rank alternatives in terms of their rightness, and Condorcet to find the right ranking. This difference carries over to the epistemic scenario (where we allow for talk about decisions in terms of "true" and "false"), with Condorcet searching for rankings with maximal likelihood, and Borda ranking alternatives by their likelihood (provided p is close to 1/2). Borda has his counterpart to the Jury Theorem reflecting this difference and regards arguments drawing on the theorem as non-starters.

Now consider Maximization. Borda maximizes *agreement among rankings*, not *acts of self-determination.* Having his own maximandum, Borda fails to be convinced by Maximization. So as far as decision rules for aggregating rankings are concerned, the Condorcet and Borda Count are on a par. Neither proposal has conclusive arguments against the other, whereas they both turn out to be reasonable rules. Yet what arguments can be made on behalf of either will be question-begging vis-à-vis methods aggregating other than ordinal rankings, and vis-à-vis methods, like fair division, that propose to make decisions in ways not involving aggregation. While of course there is more to preference aggregation than Condorcet and Borda, these considerations by themselves, properly completed, show that the multiplicity thesis holds for such aggregation.

Let me conclude this discussion with a brief reference to [1]. Brams and Sanver make an argument from the point of view of approval voting that throws light on (and can be taken to support) the multiplicity thesis for preference aggregation. They demonstrate that the outcomes of virtually all voting systems that have ever been proposed (including Borda and Condorcet methods) can be reconstructed as outcomes of approval voting for some sincere strategies. On this basis Brams and Sanver argue that these outcomes should be considered acceptable. I take this to be a friendly amendment to my argument for the multiplicity thesis for preference aggregation. My own argument is meant to show that there is a substantive case for both Borda and Condorcet and that neither has resources to defeat the other's arguments. Advocates of approval voting, following Brams and Sanver, find this claim well-supported from within their own approach to voting.

3 Saari's objections to the Condorcet proposal

3.1 To make his case that the Borda Count is the uniquely reasonable aggregation method when purely ordinal rankings are to be aggregated, Saari does two things: on the one hand, he objects to the Condorcet Proposal, with the goal of showing it to be implausible, and on the other hand, he presents arguments in support of the Borda Count, with the goal of showing that the Borda Count can be derived from neutral premises of the sort that anybody (including Condorcet sympathizers) should have to accept. Let us start with his objections and discuss his positive case in Section 4. I discuss three objections that I can identify in his work; the responses to the first and the second lead to the same general lesson, and the third objection is the most interesting one. Most theories are weakest where they criticize competing theories, and so Section 4 will be more central for my argument than Section 3: but in light of how the arguments are connected, it is best to start with Saari's objections to the Proposal. At any rate, I think Saari's objections articulate widespread concerns about majoritarian decision making and must be taken seriously on that account as well.

To begin with, Saari objects that since the Proposal, by construction, does not always deliver a unique solution, it does not solve the indeterminacy problem that beset majority rule. This objection may be directed either against the claim that the proposal captures majoritarian decision making, or against the claim that it is a reasonable rule. Either way, it fails. It cannot be a criterion of adequacy for a voting procedure's being a *majoritarian* method that it always deliver a unique result. There are ties in pairwise voting, and similarly, there must be room for ties at the general level. That does not necessarily mean that the group is left without a choice: it only means that majoritarian voting *all by itself* fails to make a unique recommendation. There is simply nothing more to say from a majoritarian standpoint to distinguish among rankings with maximal outcome, and it is then up to other criteria to bring about a unique result. (However, notice that the Proposal may still select a result if a sequence of pairwise votes leads to a cycle under majority rule, namely, when the relevant majorities are unequal.) Similarly, it cannot be a criterion of adequacy for a *reasonable* decision rule that it always deliver a unique result: there must be room for ties, ties that, if indeed all relevant criteria have been integrated into the process, should be broken by a random mechanism. Either way, the fact that the Condorcet Proposal does not make a unique recommendation *all by itself* does not pose a problem.

To put the point differently, we must distinguish *non-uniqueness* from *indeterminacy*. Indeterminacy is always a problem because it leaves the group without any recommendation. Non-uniqueness is not always a problem: what the respective method tries to accomplish may not entail a unique recommendation. Majority rule (understood as taking a sequence of pairwise votes) is indeterminate because sometimes *no* ranking emerges, not because *more than one* does. That is what indeterminacy *is*: absence of *any* recommendation. The Proposal asks which rankings are best supported by pairwise votes, which always leads to some recommendation, but there is no reason to expect that there will always only be one recommendation on the grounds that the proposal acknowledges. (Against my claim that majority rule as introduced in Section 1 is indeterminate one may stipulate that the group is indifferent between rankings obtained by cutting the cycle at some point. Yet doing so introduces an unsatisfactory account of indifference. For two alternatives, we speak of indifference if there is a tie: the group is indifferent because half of them want one thing, and half another. This provides a substantive account of indifference. That is different from observing that the procedure does not deliver a result and taking *that* to mean that it is indifferent among the rankings obtained by dissolving the source of the indeterminacy (say, by cutting a cycle): no substantive account of indifference is forthcoming in this way. Cyclicity is one way of bringing about indeterminacy, but does not *define* indeterminacy (cf. Saari, p 336)).

3.2 Saari also objects that the Proposal may display discontinuity phenomena. Suppose 17,000 voters maximally support rankings (A, C, B) and (B, A, C). The addition of one voter might cause the selection of (A, C, B) as the unique such ranking, whereas the addition of another voter instead triggers the selection of (B, A, C). Saari finds it “difficult to accept that a procedure searches for the ‘right ranking’ when it certifies radical reversals in the societal outcome—where a candidate drops from top to bottom ranked—with a trivial ‘one in 17,001’ data change” (p 339). We must ask again whether this objection addresses the claim that the Proposal is a distinguished majoritarian rule, or that it is a reasonable rule. Supposes it is the former. It is a feature of majoritarian decision making that, in principle (with *de*creasing likelihood for *in*creasing group size), minor changes may change the outcome. Saari’s emphasis that one candidate can drop from top to bottom by one tiny change does not complicate matters. If one acknowledges that it is in the nature of majoritarian decision making that tiny changes may make all the difference, and if one recalls that the Proposal chooses among *rankings*, no *additional* oddity arises in that way. If Saari’s objection addresses the claim that the Proposal is reasonable, similar points apply.

Crucially, it is a mistake to dismiss a decision rule by pointing to some implications where it allegedly errs without exploring whether (a) what seems like a counterintuitive outcome looks plausible from a viewpoint that accepts the respective rule and the arguments in its support; and if not, whether (b) the arguments in favor of the rule outweigh what is perceived as implausible; and if so (i.e., if (a) is affirmed), (c) whether the arguments given for that rule in the first place show indeed that it is a majoritarian rule, or a reasonable rule. Rejecting a decision rule in terms of one or several counterintuitive implication(s) without going through such an argument is a (quite common) methodological fallacy, an error that we might call the *fallacy of the overestimation of allegedly counterintuitive consequences.* What this means here is that anybody who finds the arguments for the Condorcet Proposal convincing to begin with will either not find it detrimental or counterintuitive that it sometimes displays this sort of discontinuity phenomena—or if she does, will want such examples to be balanced against the argumentative support for the Proposal. I submit that this is actually a rather wide-spread fallacy in reasoning about social choice.

Defenders of Borda, needless to say, are fully entitled to using precisely this move to rebut critics. For instance, a well-known objection to the Borda Count is that it is open to a certain kind of agenda manipulation. That is, suppose a number of voters must rank *A*, *B*, *C*, and *D*, and suppose they rank them in that order. Suppose that, then, some additional alternative *E* is introduced. It is possible that voters end up ranking *E* in such a way that *B* overtakes *A*. (Cf. [3], p 294, for an example). However, as Michael Dummett explains:

> The Borda count can be seen as a rough means of weighting preferences, judging the strength of a voter's preference for one option over another by the number of other options intervening between the first and the second on his preference scale. The more options the voters are asked to choose between, the less crude is the Borda count as a device for measuring strength of preference. It is therefore intrinsic to it that the introduction of new options will tend to alter the outcome. To declare this a fatal defect is in effect to argue that, so far as possible, majority preferences should go to decide the outcome. ([3], p 291)

So Dummett points out to potential critics that the manipulation charge will not impress those who are defenders of Borda on different grounds already as much as it seems to impress the critics. (It still does impress those defenders: Dummett himself goes on to make proposals for how to deal with some problems connected to this manipulation charge.) The point is the same: one cannot complain that an aggregation method errs *somewhere* but must take a more comprehensive look at what motivates it in the first place. I take it, then, that among the advocates of the Borda Count, Dummett (unlike Saari, as it seems) agrees with the approach to "reasoning about social choice" that I develop in this study.

3.3 Let us discuss a third objection that Saari raises, which draws on [16]. Suppose 800 voters must respond to an increase of students in a school. One response is to compensate teachers for larger classes; another is to hire teachers without enlarging classes. One hundred fifty voters (the Deniers) favor salary and hiring freezes while enlarging classes. Six hundred fifty wish to help the teachers, but do not want to increase salaries and hire new teachers. Three hundred of those (the Raisers) favor raises with increased class sizes, and 350 (the Hirers) favor hires with fixed class size. Voters must choose between *A*={no raises} and *B*={raises}, and between *C*={larger classes, no hires} and *D*={keep size, hires}. *A* beats *B* with 500 to 300 votes and *C* beats *D* with 450 to 350: the 150 get their way. Saari claims this is the outcome favored by a version of the Proposal modified to handle this setting (p 339), and takes this to show that the Proposal fails to *track the right rankings.*

However, closer analysis shows that this situation does not give rise to an objection to the Proposal. Surely *A* and *C* do not capture the "will of the voters," whatever that is. The problem is that the situation is hopelessly *underdescribed:* neither preferences nor alternatives are sufficiently specified for the Proposal to apply. Three questions are at stake, and each of the three positions takes stances on all three: (a) Should salaries be raised? (b) Should classes be enlarged? (c) Should teachers be hired? In what I trust is obvious notation, the Deniers' view is (no, yes, no), the Hirers' is (no, no, yes), and the Raisers' is (yes, yes, no). *A* beats *B* because Deniers and Hirers join forces against the Raisers, but only because *A* and *B* are underdescribed: choosing between *A* and *B* is choosing between (no, *blank, blank*) and (yes, *blank, blank*). It is unsurprising that the outcome is distorted if the alternatives only partially describe voters' views.

But even if the *alternatives* were fully specified, the Proposal would not be applicable. The Deniers (e.g.) cannot say whether they prefer "hiring teachers while freezing class sizes and salaries" to "enlarging classes with raises but without hires." We must also fully specify the voters' views. The Proposal, placed in ideal theory, requires a *complete description* of the problem that specifies each position in terms of a ranking of the three possible views *and* asks voters about completely specified alternatives. Suppose no other positions are considered: we disregard (say) the view that salaries should be raised, but class sizes should remain fixed, and no teachers should be hired. The Deniers rank (no, yes, no) first, and then split. Suppose 110 Raise-Inclined Deniers rank (yes, yes, no) second, and (no, no, yes) third, and the remaining 40 Hiring-Inclined Deniers rank (no, no, yes) second, and (yes, yes, no) last. Similarly, the Hirers split into 140 Raise-Inclined Hirers ranking (no, no, yes) first, (yes, yes, no) second, and (no, yes, no) last, and 210 Denial-Inclined Hirers ranking (no, no, yes) first, (no, yes, no) second, and (yes, yes, no) third. Finally, the Raisers split into 100 Denial-Inclined Raisers ranking (yes, yes, no) first, (no, yes, no) second, and (no, no, yes) third, and 200 Hiring-Inclined Raisers ranking (yes, yes, no) first, (no, no, yes) second, and (no, yes, no) third.

Only now does the Proposal apply. There are six possible rankings and three pairwise votes. The Proposal selects the ranking putting (yes, yes, no) first, (no, no, yes) second, and (no, yes, no) third: the Hiring-Inclined Raisers win. Since most voters want to help the teachers without incurring double expenses, this outcome has a good claim to capturing the "will of the people." Saari, I hope, would agree. The same conclusion emerges if we make other stipulations on how to complete Saari's case: once the problem is described properly, the Proposal does just fine.

Saari concludes that "we must worry whether procedures based on simple majority votes—in particular the Condorcet Proposal—distort outcomes by inheriting and reflecting this loss of information about the voters' wishes" (p 340). In Section 4, we discuss an objection to reliance on pairwise votes in detail, but for the time being, let us record that *this* distortion arises due to Saari's set-up, not due to the malfunctioning of the Condorcet Proposal, or through the reliance on pairwise votes. Saari, it seems, commits a "collectivized" version of what Joyce (1999) calls "the single most common fallacy people commit in the application of decision theory" (p 52). This mistake is the underspecification of outcomes, and a parallel in voting scenarios is the underspecification of voters' views or alternatives.

To be sure: practical considerations often demand partial descriptions, and it is important to explore what distortions occur in that way. Yet such scenarios do not threaten claims about ideal theory. Saari's is a wonderful case study in which the Condorcet Proposal can serve as a blueprint to assess just how far such simplified methods (no matter how advisable from a practical point of view) distort the "will of the people" to the extent that that will is captured by majoritarian methods. None of Saari's objections, then, succeed. While this says nothing about the Borda Count, the Condorcet Proposal stands undisputed. If Saari want to show that the Borda Count emerges as the uniquely reasonable aggregation rule if ordinal rankings are aggregated, he will have to make a positive case for his method of a sort that makes us abandon the Condorcet Proposal. However, as I argue in the next section, Saari has no such case: the Count cannot beat the Marquis (nor, again, can the Marquis beat the Count).

4 Arguing for the Borda count

4.1 Saari begins his case for Borda by formulating two seemingly innocuous "neutrality criteria." Accepting both commits us to Borda. Specifically, Saari defines sets of rankings whose removal from the group should not affect the outcome since those sets constitute a tie:

> To illustrate the basic idea, suppose in a two-person comparison that Sally has forty-five supporters while Bill has forty. A way to determine the will of this group is to combine in pairs a Sally supporter with a Bill supporter. Each of these forty pairs defines a tie; the aggregate tie from the forty pairs is broken in Sally's favor because there are five remaining people who support her. Thus information from the profile identifies Sally as representing the will of these people. A way to extract information about voter preferences from a profile, then, is to understand which combinations of preferences define ties. (p 342)

This job is done by his neutrality criteria. The first is the Neutral Reversal Requirement (NRR). Call two rankings "opposing" if they rank any two alternatives in reverse order. (Example: (A, B, C) and (C, B, A).) NRR stipulates that voting results remain unchanged when such rankings are removed. If two people disagree about each issue, the group choice should not change if they leave. The second condition is the Neutral Condorcet Requirement (NCR). To explain, I introduce Condorcet n-tuples. "To define this configuration with the four candidates A, B, C, and D," Saari explains, "start with any ranking of them, say, (A, B, C, D). Next, move the top-ranked candidate to the bottom to obtain (B, C, D, A). Continue this process to create the four rankings (A, B, C, D), (B, C, D, A), (C, D, A, B), and (D, A, B, C), where, by construction, each candidate is ranked in each position precisely once. With three candidates, the initial ranking (C, B, A) generates the Condorcet triplet (C, B, A), (B, A, C), and (A, C, B)" (p 343). NCR stipulates that group choices remain invariant with regard to the removal of Condorcet n-tuples.

Before we discuss the plausibility of NCR, note the following. Consider tuples of the following sort: (A, B, C, D), (B, A, D, C), (D, C, B, A), and (C, D, A, B). By Saari's construction, this is not a Condorcet 4-tuple, since it cannot be generated in the manner sketched above. However, Saari's characterization that each candidate is ranked in each position precisely once still applies. My rejection of Saari's NCR will rely on considerations that do not speak to tuples of this sort. (To wit, such tuples treat pairs of alternatives symmetrically in ways in which Saari's Condorcet n-tuples do not.) However, this is no problem, given Saari's account of Condorcet n-tuples. It just so happens that Saari's characterization in terms of each candidate being ranked precisely once in each position does not merely apply to Condorcet n-tuples.

The plausibility of NCR is crucial for Saari's discussion. Acceptance of NCR not only leads straight to the Borda Count, but also allows Saari to analyze the alleged flaws of other decision rules. NCR is a central analytical tool for [14]: Saari uses it to provide an illuminating analysis of how other voting methods go astray because they violate NCR in various ways. Nevertheless, what we are to make of that analysis turns on what sort of argument Saari can offer for NCR. The rationale for NCR is that "[s]ince the construction ranks each candidate in each position precisely once, no candidate has an advantage over any other candidate" (p 343). This argument seems to draw on *fairness to candidates.* Yet it is puzzling how such fairness bears on assessing which voters can be removed without affecting "the will of the people." (Recall that Saari is here, after all, concerned to identify which sets of preferences can be removed from the pool without changing the outcomes that an adequate method should select. That is, he is concerned to identify sets of preferences or voters whose removal from the pool would leave the "will of the people" unchanged.) An argument showing that removing Condorcet n-tuples fails to affect the "will of the people"—and that therefore adequate decision procedures must leave the group decision invariant if Condorcet n-tuples are removed—must turn on that will. Most plausibly, such an argument stresses that no information about the relative standing of candidates in rankings is lost if we remove Condorcet n-tuples. (For a discussion of fairness to voters versus fairness to outcomes/candidates, cf. [2], pp 173–4 and 255–6).

4.2 I agree that NRR captures an important aspect of neutrality. As is easy to verify, the Proposal satisfies NRR, and so NRR creates no contrast between Condorcet and Borda. That does not mean, however, that it is entirely uncontroversial. One may say that, for instance (A, B, C) and (C, B, A) should not be removed because their presence communicates that there is somebody for whom C is rather bad, and somebody for whom A is. So one might say that the joint presence of these two rankings speaks in support of B, but that this piece of information will be lost if both rankings are removed. We will, however, not pursue this line of reasoning since it does not bear on our current debate.

But what about NCR? Suppose the Condorcet triplet (C, B, A), (A, C, B), and (B, A, C) is removed. Looking at the situation from the standpoint of disagreements about pairs, we notice that the view "A is preferred to B" loses one vote; "B is preferred to A" loses two; "A is preferred to C" loses one, "C is preferred to A" loses one, "B is preferred to C" loses one, and "C is preferred to B" loses two. Three positions lose two votes, and three lose one. For each pair, it is always one view (say, "C is preferred to B") that loses two votes, whereas the opposing view ("B is preferred to C") loses one. NCR is not neutral with regard to disagreements about pairs. This is different for NRR, where each position loses one vote.

We have gathered two observations about NCR's alleged neutrality: as far as information about relative standing in rankings is concerned, NCR captures an aspect of neutrality, but as far as impact on pairwise disagreements is concerned, it does not. Reflection on pairs, instead, leads to a different neutrality criterion, which we will call the Neutral Balance Requirement. While it will turn out that, like NRR, both the Condorcet Proposal and the Borda Count satisfy that criterion, it will be useful in preparation for the main point of this subsection to discuss it. Suppose we have a set of rankings $M=\{R_1, ..., R_l\}$, for a natural number l. Consider then the set PM=$\{(A, B)_i$: A and B are alternatives, and there exists a ranking R_i, $1 \leq i \leq l$, in M such that A is ranked ahead of B in $R_i\}$. For instance, if M consists of the two rankings $R_1=(A, B, C)$ and $R_2=(C, A, B)$, then PM=$\{(A, B)_1, (A, C)_1, (B, C)_1, (C, A)_2, (C, B)_2, (A, B)_2\}$. The purpose of the indexing is to make sure that, in this case, the pair (A, B) is counted twice, since A is ranked ahead of B both in (A, B, C) and in (C, A, B), and in general to make sure that each pair is counted as many time as it appears in that order in some ranking in M. Call a set of rankings $M=\{R_1, \ldots, R_l\}$ *balanced* if there exist sets $\mathrm{PM}_1, \ldots, \mathrm{PM}_m$, for a natural number m, such that (a) PM is the union over all PM_i, $1 \leq i \leq m$; (b) the intersection of any PM_i and PM_j, $1 \leq i \neq j \leq m$ is empty; and (c) for all such i, $\mathrm{PM}_i=\{(A, B)_j, (B, A)_k$, for some alternatives A and B and some rankings R_i and R_k in $M\}$. (Notice that, to keep the notation simple, "A" and "B" sometimes denote fixed alternatives and sometimes function as variables ranging over alternatives; it should be clear, in each case, what is meant.)

A set of rankings M is balanced, that is, if for any pair (A, B) that occurs anywhere in rankings in M, the opposing pair (B, A) also occurs in some ranking in M, and the set PM of all pairs that occur anywhere in some ranking in M (which by construction lists all pairs as many times as they occur in rankings in M) is a disjoint union over pairs (A, B) and their opposing pairs (B, A). For an example, consider $M=\{(A, B, C, D), (C, B, A, D), (A, D, C, B), (D, B, C, A), (B, D, A, C), (C, D, A, B)\}$. To see that this is a balanced set, note that (A, B, C, D) contributes the ordered pairs (A, B), (A, C), (A, D), (B, C), (B, D), and (C, D) to PM, (C, B, A, D) the pairs (C, B), (C, A), (C, D), (B, C), (B, D), and (C, D)), and so on. (B, D), for instance, occurs twice already, and the indexing makes sure that (B, D) occurs in PM as many times as it occurs in any ranking in M. Once the reader has constructed PM in this manner, it will be easy to see that PM is a disjoint union over sets including only a pair and its opposite (such as (A, B) and (B, A)). As opposed to that, the set $\{(A, B, C, D), (A, D, C, B), (B, C, A, D)\}$ is not balanced: the ranking (A, B, C, D) ranks A ahead of D, but there is no ranking in this set that ranks D ahead of A.) The set $M=\{(A, B, C, D), (C, B, D, A), (B, D, C, A), (D, B, C, A)\}$ is not balanced either: though it is true that for each pair that is ranked somewhere the opposing pair also appears in M, the pair (C, A) occurs three times in PM, but (A, C) occurs only once. Opposing rankings form balanced sets, but there are balanced sets free from opposing rankings (if there are more than three alternatives—the set M just discussed is an example).

The Neutral Balance Requirement (NBR) now stipulates that the group choice remain unchanged if balanced sets are removed. As I said above, we are led to this criterion by reflecting on pairs, so the prima facie rationale is that removing such sets does not affect the strength of views on the relative standing of pairs vis-à-vis the opposing pairs. It turns out, however, that both the Condorcet Proposal and the Borda Count satisfy NBR. To see that the Proposal does, note that, by construction, each possible ranking of the m options involved will lose an equal number of pairwise votes if a balanced set of rankings is removed from the original pool of rankings. To see that the Borda Count does as well, note that just as each possible ranking loses the same amount of support, so does each alternative. So we find that the Condorcet Proposal satisfies NRR and NBR, *but not* NCR, whereas Borda satisfies NRR, NBR, *and* NCR.

The crucial question now becomes: why should we plausibly impose NCR *in addition* to NRR and NBR and hence impose a criterion that rules out the Condorcet Proposal and leads straight to the Borda Count? Saari, of course, would not want us to answer this question by assessing NCR from the standpoint *of how it affects pairs* (as we started to do above) but he has not yet delivered arguments that keep us from doing so. *Pace* Saari, the school example in 3.3 fails to discredit the use of information contained in pairs. We explore what I take to be Saari's main argument "against pairs" in 4.4 (which is an argument he has championed throughout his recent writings) and show that it fails as well. So at this stage Saari must explain why we should adopt NCR in addition to NBR without simply assuming that criteria in terms of pairs have already been discredited. They have not.

Crucially, now, such an explanation is unavailable without endorsing commitments vis-à-vis the purposes of the aggregation. That is, such an explanation will include statements like "criterion X should be adopted because the purpose of aggregating rankings is such and such," or "X should be adopted because in aggregating rankings we are concerned to do such and such." Suppose somebody finds NRR and NBR persuasive because the sorts of removal of rankings licensed by these two criteria leave invariant the relative standing of pairs vis-à-vis their opposites. Asked to elaborate, this person would say that she thinks the purpose of the aggregation is to assess how strongly rankings are supported by pairwise votes, and if rankings are removed in such a manner that for each pair (A, B) that loses the support of one voter, so does its opposing pair (B, A), then the support for each possible ranking that could be the group choice will go down by the same amount. For somebody with such a view, NCR would be unappealing, because it is not true for NCR that it removes pairs under such conditions. By reasoning about these criteria as suggested in this paragraph, of course, one commits oneself to views about aggregation that are embodied by the Condorcet Proposal.

In support of the view that NCR should be adopted in addition to NRR and NBR, however, Saari may now say that NCR is persuasive because no information about the relative standing of candidates in the rankings vanishes if we remove a Condorcet n-tuple. (Recall, however, that this argument is my reformulation of what Saari said by way of giving a rationale for NCR. His own rationale was in terms of fairness to candidates, but, as I have pointed out above, such fairness has nothing do with the will of the people.) In fact, Saari may add that arguments of this sort also speak in favor of NRR and NBR: both criteria leave the relative standing of all candidates unchanged. (So, indeed, there is a convergence of arguments for NRR and NBR, as sketched in this and preceding paragraph: but crucially, there is *no* such convergence of arguments for NCR.) Yet to argue against the view that we sketched in the preceding paragraph (as he must in order to have a case for NCR), Saari would now have to say that the purpose of aggregating rankings is to assess such relative standing of candidates and then to rank them accordingly. And that is precisely what the Borda Count does: it asks about the support for each of the m options in all rankings.

The main insight we have gained through this discussion is that by justifying his criteria, and especially by explaining why in addition to NRR and NBR one *also* should endorse NCR, Saari must give reasons unavailable to the impartial position from which he means to "assess the data." Instead, he must endorse commitments to the purpose of aggregation—commitments that will be plausible to some, but not to others, and that, crucially, cannot themselves be justified simply by reference to "the data." Thus Saari fails to identify a pre-theoretical standpoint from where to formulate neutrality criteria of the desired sort. I doubt there is such a standpoint: there is no neutral way of looking at "the data," free from commitments to the purposes of the aggregation, that will provide us with intuitively obvious invariance criteria that in turn lead to one and only one distinguished aggregation method. Every look at "the data" will be a look "from somewhere," that is, will be theory-laden, in a sense just explained. The neutral view of "the data" is a view from nowhere.

4.3 I submit the following account of the situation. Both the Condorcet Proposal and the Borda Count represent robust views on aggregation: Each is formulated around an elementary and plausible idea about group decision making, and each is supported by a set of strong arguments. Each has counterintuitive implications from the point of view of the other, perhaps even some from its own point of view. Each view conforms to certain neutrality criteria, and can formulate reasons (capturing the core idea of the respective proposal) justifying why those criteria are appropriate, and others are not. In particular, reasoning conducive to the Condorcet Proposal supports NRR and NBR, but not NCR, whereas reasoning conducive to the Borda Count supports NRR, NBR, and NCR. It will be fruitless to press on either method to reveal implications that look odd from the point of view of the other, but that (a) do not look implausible from the point of view of that rule, or (b) that will, on balance, not persuade a defender of that rule to abandon her position. I have called such a move the fallacy of the *overestimation of allegedly counterintuitive consequences.* Now we have identified a symmetric error, which I call the fallacy of the *overestimation of allegedly independently plausible axioms*. I call this a fallacy because it is fruitless to show that a decision rule does not conform to conditions that look plausible only from the point of view of the other. Saari seems to commit that error as well, by criticizing the Proposal for not abiding by NCR although it is quite straightforward, from the ideas that guide the Proposal, that it indeed should not abide by NCR. (Note that, in his [14], Saari explicitly distances himself from an approach that he there calls "axiomatic." However, what he means there is approaches that specify "measures for election and decision methods and identif[y] which procedures maximize the measure" (p 341). What I mean here is, more generally, the stipulation of any kind of condition postulated *ex ante*, based on plausibility considerations of whatever sort and independently of inquiries into the features of specific methods. What Saari rejects there under the title "axiomatic approaches" I reject as well: cf [7], p 721. The issue here is a very different one).

Both fallacies are based on the same pattern: they identify one or several facts about a decision rule (such as an implication given a certain set of rankings, or its lack of consistency with a certain axiom), and declare that those cause devastating problems for that rule, without going through a careful investigation of whether defenders of that rule have resources to address such claims. Put differently, both of these fallacies identify allegedly troublesome features of a decision rule and then assign the claim that these features prove fatal such an enormous degree of certainty that all arguments in support of that rule become obsolete. Yet once such reasoning is blemished as erroneous, we must explain what would actually count as a *successful* objection to a decision rule.

So what does count as a successful objection against decision rules? The sketch I gave in 3.1, at the end of my response to Saari's second objection, suggests an answer. Suppose we find some apparently counterintuitive implication of a decision rule, or a conflict with an apparently plausible axiom. Then we should check whether this implication or conflict is plausible on the terms adopted by defenders of that rule. If so, an opponent must engage arguments in support of that rule to explore whether the standpoint from which such implications or conflicts appear plausible can itself be supported. If these arguments succeed, the objections fail: they merely spell out consequences of that rule. If these arguments fail, nothing is gained if counterintuitive implications or conflicts with axioms are acceptable to somebody endorsing the rule. The rule would have to be withdrawn. If, however, we find that the counterintuitive implication or the conflict are unacceptable to defenders of the rule to begin with, we must still explore whether, on balance, the arguments in support of the rule outweigh whatever negative import the objections may have. This will once more turn on an investigation of the arguments. Without undertaking such an investigation, one cannot balk at implications that depend on those arguments for every bit of their plausibility. Intuitions about reasonableness in group decision making are too amorphous to allow for the quasi-foundationalist isolation of some selective features of a decision rule, only to classify them as counterintuitive, and to take that judgment all by itself to outweigh, without further investigation, what may be said in favor of the decision rule. The required (*coherence*-focused, rather than *foundations*-focused) investigations are much more heavy-handed. But this is simply what is needed for investigations into questions of group rationality.

4.4 Yet Saari offers another argument for NCR, which is also an argument against the reliance on pairwise votes that has figured prominent in Saari's recent work: see [13], chapter 5, section 1, [15], in particular sections 6 and 8, and [16]; see also [13], section 3.3.3 and [9], sections 3.1 and 3.2. Still, Saari's argument, as I shall argue here, is a non-starter against the Condorcet Proposal. Whatever its use, it cannot help Saari make his case against the Proposal.

Consider the Condorcet triplet (A, B, C), (C, A, B), and (B, C, A). Those rankings contribute to the selection of the rankings championed by the Proposal by contributing information about how many individuals have which preference between which alternatives: two individuals support "A over B," one "B over A," two "B over C," one "C over B," two "C over A," and one "A over C." Next consider all profiles generating that distribution of votes over those pairs (which Saari calls "parts" of the Condorcet configuration). One of them is the Condorcet triplet. Another includes (A, B, C) and (C, B, A), while the third voter has cyclical preferences, ranking A over B, B over C, and C over A. Pairwise votes among those people contribute in the same way to the selection of the rankings championed by the Proposal as the Condorcet triplet. Yet in this case, so Saari says, the natural outcome for the group is the cyclical preference structure: (A, B, C) and (C, B, A) cancel out due to NRR, leaving the third person to determine the outcome. The Proposal delivers the wrong result. According to Saari, 80% of all profiles that can be constructed in this way (generate the same distribution of votes over pairs) support cyclical social outcomes. As Saari says (p. 347), quoting himself, and discussing both pairwise majority rule and the Condorcert Proposal in one breath,

> [t]hese numbers capture a sense of Saari's argument (...) that the majority vote statistically interprets the parts of a Condorcet *n*-tuple as coming from profiles consisting of cyclic voters where the indeterminate cyclic outcome is an appropriate conclusion. Rather than a natural tie, the majority vote introduces a cycle because it is trying to meet the needs of nonexistent cyclic voters! As Saari has argued, the indeterminacy problem and all difficulties where the majority vote and Condorcet Proposal distort the wishes of the voters arise only because the majority vote mistakenly interprets the Condorcet configuration as being the contribution of nonexistent votes with cyclic preferences.

Or, to let Saari state his argument in a different way, this is what Saari and Merlin ([15], p 421) say to make the same point:

> These cycles occur because the pairwise vote cannot distinguish the Condorcet profile (of transitive preferences) from ballots cast by irrational voters with cyclic preferences (...). In other words, using the pairwise vote with a Condorcet profile differential has the effect of dismissing, for all practical purposes, the crucial assumption that the voters are rational. Instead (...), the pairwise vote treats the Condorcet *n*-tuple (...) as though the votes are cast by non-existent, irrational voters.

Since this is dense, and, crucially, applies differently to majority rule and the Condorcet Proposal (since the latter does not produce any cycles) let me explain what I think is going on. Suppose three persons must rank *A*, *B*, and *C*. Suppose, again, that two support "*A* over *B*," one supports "*B* over *A*," two "*B* over *C*," one "*C* over *B*," two "*C* over *A*," and one "*A* over *C*." Different remarks apply if we use majority rule as introduced in Section 1, or the Proposal. In fact, Saari's complaint speaks most directly to majority rule, and less so to the Proposal, but we can use some extrapolation to apply the complaint to the Proposal. Suppose we use majority rule. Then a cycle emerges, which Saari thinks is appropriate in 80% of cases, but in the presence of a Condorcet triplet, the intuitive outcome is a tie (as Borda delivers it). No tie emerges under majority rule because "the majority vote mistakenly interprets the Condorcet configuration as being the contribution of nonexistent votes with cyclic preferences." Suppose we use the Proposal. Then, in all those cases in which a cycle is the natural outcome, the Proposal gets it wrong because it insists on a ranking as a solution. And in all those cases in which a tie is the appropriate solution (and this is where we have to extrapolate a bit) the problem seems to be that the Proposal delivers a tie among *rankings*, rather than, as Borda would, a tie among *outcomes*. (Saari's objections to majoritarian decision making across his writings seem to be directed against majority rule, but in [14] he seems to apply the same point to the Condorcet proposal without paying much attention to the differences.)

As far as majority rule is concerned, I think Saari's objection succeeds. This is yet another way of seeing that that rule as defined Section 1 is *not* what we should mean by majoritarian decision making in the general case, so in this sense Saari's reflections are grist for the Condorcet mill. I have, at this stage, nothing to say to Saari's objection that the Condorcet Proposal delivers a tie among rankings rather than among alternatives: earlier parts of this study, I think, have assembled arguments showing that an approach to group-decision making in terms of selecting a ranking is coherent and defensible. So in what follows I will only be concerned with that bit of Saari's objection that worries that in certain cases a ranking is chosen even though a cycle is the natural solution. I will show that Saari has no successful objection to the proposal.

The source of trouble that Saari identifies is that the Proposal still delivers results if preferences are intransitive, but fails to be intuitively reasonable if they are. Thus Saari's argument challenges the applicability of the Proposal if we have reason to think that voters have irrational (non-transitive) preferences. That may happen in two ways. Either (a) we know the preferences and know that some are cyclical, or (b) we only know the results of pairwise voting and it is likely that the relevant pairwise results derive from cyclical preferences. The Proposal, again, captures majoritarian decision making *in ideal theory*. What characterizes such ideal theory is, in particular, that we do not worry about availability of information about preferences: (b) *never holds*. Saari's considerations are worrisome only if (a) holds, that is, only if some voters have intransitive preferences. Nothing so far discredits the claim that the Proposal captures majoritarian decision making (and is a reasonable rule) *for rational voters* wishing to be collectively rational. In ideal theory, we know whether voters have intransitive preferences: that is part of what ideal theory *is*.

Suppose we leave ideal theory and wonder which method to adopt in concrete settings. Our input is only what voters reveal. For such settings, Saari teaches an important lesson, namely, that the Proposal might deliver implausible results if some have cyclical preferences. If there are such preferences, cyclical group outcomes should not be by definition excluded as collective outcomes, as the Proposal does. We should worry if, as condition (b) stipulates, voting results are best explained by the presence of irrational voters. One simple way of dealing with that problem is to ask voters to submit rankings rather than pairwise votes. This is legitimate given that Borda *must* ask for rankings. Also, the fact that the Proposal uses only pairwise votes has no methodological or epistemological virtues that would be undermined if we asked voters for rankings. So while Saari offers valuable insights about problems that may arise when the Proposal is put into practice, no objection is forthcoming here. On the contrary: Saari's findings can be readily integrated into the framework of the Condorcet Proposal.

4.5 Let me approach the matter from a different angle, which, I hope, will make crystal-clear why Saari's argument fails as an objection, or as part of an objection, to the Condorcet Proposal. If the dispute is between Condorcet and Borda, Condorcet is entitled simply to assume rational voters (voters having rankings) because Borda *by definition presupposes rankings*. So in the case of ideal-theory, where rational voters want a rational group outcome, Saari's argument does not apply. As opposed to that, non-ideal theory assesses the case of voters without rankings: Borda, by definition, does not apply there, and what Saari does is best understood as showing why one should not use Condorcet either. (If individuals are, for whatever reason, incapable of ranking the alternatives, one should not be surprised that there is not straightforward way in which the group may rationally do so: measures would have to be taken to put individuals in a position to come up with rankings.) But this does not bear at all on the dispute between Borda and Condorcet in ideal theory: it does not reveal anything about their relative advantages *because the domain it speaks to is one for which Borda is undefined*, and so Saari's argument cannot possibly support the Borda Count over the Condorcet Proposal.

Put differently: Saari, as a defender of the Borda Count, is in no position to criticize the Condorcet Proposal for *not actually using* transitivity of preferences, or for not recognizing it, because the only circumstances under which this could be a problem are circumstances under which Borda does not even apply. Saari's argument, then, as insightful as it is in what it reveals about non-ideal theory, and as useful as it is to highlight once again why we should not think of majoritarian decision making in terms of majority rule as defined in Section 1, is a non-starter as an argument against the Condorcet Proposal. Saari has no case against the Proposal, and he has no case in support of the Borda Count that would compel defenders of the Proposal to change their views. (To remind the reader: I believe that the converse is true as well: defenders of the Condorcet Proposal also have no case in support of their method that would force theorists like Saari to change *their* minds.) The Count cannot beat the Marquis, then, and the multiplicity thesis as defined in the introduction stands unrefuted.

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