Math 464/564 Midterm Exam Review Sheet - October 10, 2017

General Representation Theory Material

- Basic group theory, symmetric, dihedral and general linear groups, subgroups and cosets, Lagrange's theorem.
- Group actions, orbits, stabilzers. "Orbit-stabilizer" theorem, applied to conjugacy classes and centralizers.
- Basic symmetric group material, cycle type, centralizer structure, multiplying permutations, conjugacy classes in S_n, conjugacy class sizes in S_n.
- Definitions of matrix representation of a group G and of a module for the group algebra kG. Equivalence between these two ideas. Trivial representation.
- Group acting on a set X and the corresponding permutation module. Computing the character of this module. Special case where G acts on left cosets of H, and equivalence of this module with $\mathrm{Ind}_H^G 1$. Even more special case of the regular representation. Decomposing the regular representation into a direct sum of irreducibles.
- Submodules, irreducible modules, complete reducibility. Definition of a G-invariant inner product. Understand the proof of Maschke's theorem and why it fails sometimes in prime characteristic. Definition of cyclic module.
- Definition of a module homomorphism, Schur's Lemma.
- Commutant algebra of a matrix representation X and why it is the same as $\mathrm{End}_G(V)$ for the corresponding module V. Some understanding of how we counted the number of irreducible $\mathbb{C}G$ modules using the center of an endomorphism algebra.
- Definition of characters. Linear characters, degree of a character, irreducible characters. Class functions, inner products. Character table. Orthogonality of matrix functions. Row and Column Orthogonality. Irreducible characters as a basis of class functions. Group algebra as a direct sum of matrix algebras.
- Given a character table, use it to find orders of elements, order of the group, center of the group, kernels of irreducibles, to decompose other characters into irreducibles.
- Operations on modules: Direct sum tensor product, restriction and induction. What are the corresponding characters for these operations? Calculating induced characters directly ((1.28) in the book). Irreducible characters for direct product $G \times H$ of groups.
- Frobenius reciprocity.

Symmetric Group Material

- Definition of partitions and compositions. Young subgroups S_λ. Tableau and tabloids and how λ-tabloids represent cosets of S_λ, and thus form a basis of the permutation module M^λ.
- Row and column subgroups. Definition of κ_t and polytabloids $e_t = \kappa_t\{t\}$. Total and partial orders. Lex and dominance orders on partitions. Partial order on λ-tabloids.
- Definition of Specht modules as the span of polytabloids. Fundamental combinatorial lemma. Basis of Specht module in terms of standard polytabloids. You do not need to understand how to construct Garnir elements but should at least have some idea how we determined the basis of S^λ.
- James submodule theorem.
- λ tableau of type μ. Homomorphisms between S^λ and M^μ. Semistandard basis indexed by semistandard tableaux.
- Branching rule for induction and restriction, including characteristic free version of the restriction one proved in class.
- Young's rule
- You should be able to construct a basis for any S^λ and, given time, construct matrices for the corresponding representation.