

## Math 464/564 Fall 2017 Homework Number 2- Due 9/12/17

1. The dihedral group  $D_8$  acts on  $\mathbb{R}^2$  naturally, where  $r$  acts as a 90 degree counterclockwise rotation and  $s$  acts as a reflection across the line  $y = x$ . Write the matrices for  $r$  and  $s$  in the standard basis.

- Explain geometrically why this representation is irreducible over  $\mathbb{R}$ .
- Is the same true over  $\mathbb{C}$ ? Explain.

2. Give an example if an infinite group  $G$ , different from the one in the book, and a finite-dimensional  $\mathbb{C}G$  module where Maschke's Theorem fails.

3. Page 48 # 1.

4. Page 49 #6.

5. (James-Liebeck p. 52) Define permutations  $a, b, c \in S_6$  by:

$$a = (123), b = (456), c = (23)(45)$$

Let  $G = \langle a, b, c \rangle$ , the subgroup generated by  $\{a, b, c\}$ .

a) Check that

$$\begin{aligned} a^3 = b^3 = c^2 = 1, ab = ba \\ c^{-1}ac = a^{-1}, c^{-1}bc = b^{-1} \end{aligned}$$

Deduce that  $G$  has order 18.

b) Suppose that  $\epsilon$  and  $\mu$  are complex cube roots of unity. Prove that there is a representation  $\rho$  of  $G$  over  $\mathbb{C}$  such that:

$$\rho(a) = \begin{pmatrix} \epsilon & 0 \\ 0 & \epsilon^{-1} \end{pmatrix}, \rho(b) = \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix}, \rho(c) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

For which values of  $\epsilon, \mu$  is  $\rho$  faithful? For which values is it irreducible?

6. Let  $G = S_4$  and let  $H = \{e, (12)(34), (13)(24), (14)(23)\}$ .

a) Write down the six left cosets of  $H$ .

b) Let  $G$  act by left multiplication on these six right cosets, giving six-dimensional permutation module  $M$ . Find the matrices for the action of  $(12)$  and  $(123456)$ . Is  $M$  irreducible? Explain.

c) Write down the matrices for  $(12)(34)$ ,  $(13)(24)$  and  $(14)(23)$ . Explain your answer.