

**Math 464/564 Fall 2017 Homework Number 12- Due Thursday 12/7/17**

1. Sagan Chapter 4.11 # 6
2. Sagan Chapter 4.11 # 15
3. Expand  $s_{(2,1)}s_{(3,2)}$  in terms of Schur functions. Show your work.
4. Use the Murnaghan-Nakayama rule to calculate  $\chi^{(5,3,3)}((1, 2, 3, 4, 5)(6, 7, 8)(9, 10, 11))$ .
5. Describe completely the column in the character table of  $S_n$  corresponding to the conjugacy class of permutations which are a single  $n$ -cycle.
6. Suppose  $\lambda$  and  $\mu$  are partitions of  $n$  with  $\chi^\lambda(e) = \chi^\mu(e)$ , i.e. the diagrams for  $\lambda$  and  $\mu$  have the same multiset of hook lengths. Does it follow that  $\lambda = \mu$  or  $\lambda = \mu'$ ?
7. A partition  $\lambda$  is a  $p$ -core if there are no hook lengths divisible by  $p$ .
  - a) Classify all two-core partitions.
  - b) Let  $\lambda \vdash n$ . Prove that the number of odd hook lengths minus the number of even hook lengths is a triangular number. Hint: if there is an even hook length then there must be a hook of length two. What does removing it do to the multiset of hook lengths?