Math 461/561 Assignment #4- Due Tuesday October 5, 2010

1. Let L be a Lie algebra, and let I and J be nilpotent ideals. Prove that $I + J$ is nilpotent. Use this to prove that L contains a unique maximal nilpotent ideal which contains all nilpotent ideals.

2. Prove the statement on p.40 immediately after Exercise 3: “More generally, ...”

3. Let $T : V \to V$ be a linear map. Suppose $\{v_1, v_2, \ldots, v_n\}$ are eigenvectors of T with *distinct* eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Prove $\{v_1, v_2, \ldots, v_n\}$ is a linearly independent set. Hint: It is ok to consult a linear algebra book, just make sure you understand the proof you write down.

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