1. 13.2.1B

2. 13.2.2B (Hint: The symmetries of the object in Figure 13.5 are the same as the symmetries of a square)

3. 13.2.4B

4. Suppose we want to place 4 red, two yellow and two green keys on a circular key ring. Use Burnside’s Theorem to count the number of ways to do this.

5. Find the number of different colorings of a cube with two white, one black and three red faces.

6. How many different chemical compounds can be made by attaching \(H\), \(CH_3\), \(C_2H_5\) or \(Cl\) radicals to the four bonds of a carbon atom. (The radicals lie at the vertices of a regular tetrahedron with the carbon atom in the center).

7. Give a simple proof of Cauchy’s theorem for \(p = 2\). (Hint: pair up)

8. Suppose \(H\) is a subgroup of \(G\) and \(g \in G\). Let:

\[
gHg^{-1} = \{ghg^{-1} \mid h \in H.\}
\]

a. Prove that \(gHg^{-1}\) is also a subgroup.

b. Let \(X\) be the set of all subgroups of \(G\). Prove that \(G\) acts on \(X\) by conjugation, as in part a.

c. The stabilizer of a subgroup \(H\) under this action is called the normalizer:

\[
N_G(H) = \{g \in G \mid gHg^{-1} = H.\}
\]

Let \(G = S_4\) and \(H = \langle(1,2,3,4)\rangle\) be a cyclic subgroup of order 4. Determine the normalizer of \(H\).