1. Let $\mathbf{r}(t) = (t, 2\cos t, 2\sin t), -4 \leq t \leq 4$. Find the length of the curve.

$$\mathbf{r}'(t) = \left(1, -2\sin t, 2\cos t\right)$$

$$|\mathbf{r}'(t)| = \sqrt{1 + 4\sin^2 t + 4\cos^2 t} = \sqrt{5}$$

$$\mathbf{A}_L = \int_{-4}^{4} \sqrt{5} \, dt = 8\sqrt{5}$$

2. Let $\mathbf{r}(t) = (t, t^2, e^t)$. Find an equation for the unit tangent vector $\mathbf{T}(t)$.

$$\mathbf{r}'(t) = \left(1, 2t, e^t\right)$$

$$|\mathbf{r}'(t)| = \sqrt{1 + 4t^2 + e^{2t}}$$

$$\mathbf{T}(t) = \frac{1}{\sqrt{1 + 4t^2 + e^{2t}}} \left(1, 2t, e^t\right)$$
1. The position function of a particle is given by $\vec{r}(t) = (t^2, 5t, t^2 - 2t)$. When is the speed a minimum (Hint: minimize the square)?

$$\vec{r}'(t) = (2t, 5, 2t - 2)$$

$$|\vec{r}'(t)| = \sqrt{4t^2 + 25 + 4t^2 - 8t + y}$$

So $\text{speed}^2 = 8t^2 - 8t + 25$, set derivative equal to zero

$$16t - 8 = 0$$

$t = \frac{1}{2}$

2. Find and sketch the domain of the function $f(x, y) = \sqrt{2x + 3} + \sqrt{y}$. Need $y \geq 0$ and $2x + 3 \geq 0$ so $x \geq -1.5$
1. Let \( \vec{r}(t) = (e^t, e^{2t}) \). Find the velocity, acceleration and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for \( t = 0 \).

\[
\vec{v}(t) = (e^t, 2e^{2t}) \quad \text{Speed} = |\vec{v}(t)| = \sqrt{e^{2t} + y e^{4t}}
\]

\[
\vec{a}(t) = (e^t, 4e^{2t})
\]

Note \( |\vec{v}(t)| \) lies on \( y = x^2 \) and only for \( x > 0 \)

![Graph](graph.png)

2. Let \( f(x, y) = y - 2x^2 \). Sketch and neatly label the level curves for \( k = 0, 1, 4 \).

\[
\begin{align*}
K = 0 & \quad y = 2x^2 \\
K = 1 & \quad y = 2x^2 + 1 \\
K = 4 & \quad y = 2x^2 + 4
\end{align*}
\]