Lecture 7

Rank If \( \hat{r}(t) \) is position at time \( t \) then \( \hat{r}'(t) = \hat{v}(t) \) = velocity, \( \|\hat{v}(t)\| = \text{speed} \)
\( \hat{r}''(t) = \hat{a}(t) \) = acceleration

Example Particle has init position \( \hat{r}(0) = (1,2,3) \) and init velocity \( (-1,1,2) \).
Suppose \( \hat{a}(t) = (t,2-t^3,t^3) \). Find \( \hat{r}(t) \).

Newton's 2nd Law Force \( \hat{F}(t) \) acts on object of mass \( m \) then
\( \hat{F}(t) = m \hat{a}(t) \)

Example Object moves on circle radius \( a \), angular speed \( \omega \), so
\( \hat{r}(t) = (a \cos \omega t, a \sin \omega t) \)
\( \hat{r}'(t) = (-a \omega \sin \omega t, a \omega \cos \omega t) \)
\( \hat{r}''(t) = (-a \omega^2 \cos \omega t, -a \omega^2 \sin \omega t) \) \( \hat{r}''(t) = -a \omega^2 \hat{r}(t) \)

So \( \hat{F}(t) = m \hat{a}(t) = -m a \omega^2 \hat{r}(t) \)
Centripetal Force - points back toward \( \hat{r} \)

#28 Baseball hit 3 ft above ground toward fence
10 ft high and 400 ft away.
Ball leaves bat at 115 ft/s at \( \Phi 50^\circ \) above horizontal.
Is it a home run?
Put origin at feet so \( \hat{r}(0) = (0, 3), \hat{\gamma}(0) = (115 \cos 50, 115 \sin 50) \approx (73.92, 88.10) \) \( \hat{\gamma}(t) = (0, -32) \) so \( \hat{r}(t) = (73.92, 88.10 - 32t) \) \( r(t) = (73.92t, -16t^2 + 88.10t + 3) \) Gets to wall when \( 400 = 73.92t \) so \( t = 5.41 \) \( r(5.41) = (400, 11.33) \) Home run!

13.426 Tank fires 400 m/s. What two angles of elev to hit target 3000 meters away

\( \hat{r}(t) = (0, -9.8^2 \text{ m/s}^2) \) \( \hat{v}(0) = (400 \cos \theta, 400 \sin \theta) \) \( \hat{r}(0) = (0, 0) \) \( \hat{r}(t) = (400 \cos \theta, -9.8t + 400 \sin \theta) \) \( \hat{r}(t) = (400 \cos \theta t, -9.8t^2 + 400 \sin \theta t) \) Hits ground when \(-9.8t^2 + 400 \sin \theta t = 0 \) so \( t = 0 \) or \( t = \frac{400 \sin \theta}{9.8} \)

Want \( (400 \cos \theta) \left( \frac{400}{9.8} \sin \theta \right) = 3000 \)

\[ \frac{32653}{3} \sin \theta \cos \theta = 3000 \]

\[ \sin \theta \cos \theta = 0.18375 \]

\[ 2\theta = \sin^{-1}(0.18375) \]

\[ = 10.58 \text{ or } 169.42 \]

\[ \theta = 5.3^0 \text{ or } 84.7^0 \]
Recall \( \vec{T}/r = \frac{\vec{T}'/r}{1/r} \Rightarrow \vec{V}/r = \vec{r} \)

\[
\vec{a} = \vec{V}' = \sqrt{r} \vec{T} + r \vec{T}'
\]

Recall \( r = \frac{lt''}{r} \) so \( \vec{T}' = \vec{r} \vec{v} \)

\[\vec{N} = \vec{T}'/\vec{r}\]

Resolves acceleration into tangential and normal components

\[
\vec{a} = a_T \vec{T} + a_N \vec{N}
\]

Highly Recommend reading 13.4 Kepler's Laws

1. Planet orbits sun on ellipse with sun at focus.
2. Equal areas = times
3. Square of period of revolution proportional to cube of length of major axis.
Now we consider real-valued functions of more than one variable:

\[ f(x, y, z) = x^2 \sin z - xy^3 \quad f : \mathbb{R}^3 \rightarrow \mathbb{R} \]

i.e. domain is some subset of \( \mathbb{R}^n \)

\[ f(x, y) = \sqrt{x-1} + x \sqrt{y-3} \quad \text{Domain} \]

\[ f(x, y) = \sin^{-1}(x+y) \]

\( D: -\pi \leq x+y \leq \pi \)

Graphs: Suppose \( f(x,y) \) has domain \( D \subseteq \mathbb{R}^2 \).

Graph is all points \( \{(x,y, f(x,y)) \mid (x,y) \in D \} \).

\[ f(x, y) = x^2 + 2y^2 \]

Traces \( z = \text{constant} \) are ellipses longer in \( x \) direction.
Ex. \( f(x,y) = 10 - x - y \) Graph \( z = 10 - 2x - y \) is a plane.

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Level Curves

**Def.** The level curves of \( f(x,y) \) are curves \( f(x,y) = k \)

Ex. \( f(x,y) = x^2 + 2y^2 \)

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Ex. Given \( f(x,y,z) \) you can sketch level surfaces in \( \mathbb{R}^3 \).

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Problems

14.1 #32 (visualize), #45

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