1. (10 points) Let \( f(x, y) = x^2 + 2y^2 \). Sketch level curves \( f(x, y) = k \) for \( k = 1, 2, 3, 4 \) and on the same picture sketch the vector field \( \nabla f \).

b. Find the direction and the rate of maximum increase for \( f(x, y) \) at the point \((1, 3)\).
2. **(15 points)** Let \( S \) be the surface \( z = x^2 + y^2 \) lying below the plane \( z = 2 \).

a. Neatly sketch the surface.

b. Parameterize the surface as \( \vec{r}(u, v) \) with \((u, v) \in D\). Be sure to describe the region \( D \).

c. Let \( \vec{F}(x, y, z) = (z, x, y) \). Use Stokes’ theorem to evaluate:

\[
\int \int_S \text{curl} \, \vec{F} \cdot \vec{n} \, dS.
\]
3. **(15 points)** Let $C$ be the curve of intersection of the plane $y + z = 7$ and the cylinder $x^2 + y^2 = 4$, oriented to be counterclockwise when viewed from above. Use Stokes’ theorem to compute $\int_C \vec{F} \cdot d\vec{r}$ where $F(x, y, z) = (-y^2, x, z^2)$. 
4. **(15 points)** Let \( S \) be the cylinder (including top and bottom) \( x^2 + y^2 = 1, 0 \leq z \leq 1 \), with outward pointing normal. Let \( \vec{F}(x, y, z) = (x^3, y^3, e^{-z}) \). Use the divergence theorem to compute

\[
\int \int \mathbf{F} \cdot \mathbf{n} \, dS.
\]
5. **(10 points)** Let $S$ be the portion of the paraboloid $z = x^2 + y^2$ which lies between the planes $z = 1$ and $z = 4$. Write down *but do not evaluate* an integral which gives the surface area of $S$. 
6. **(10 points)** Consider the surface parameterized by \( \vec{r}(u, v) = (u^2 + v^2, uv, uv^2) \). Find the tangent plane to this surface at the point where \( u = 1, v = 2 \). Give the equation of the tangent plane in two forms, first parameterize it. Second, give it in \( ax + by + cz = d \) form.
7. (10 points) Use Green’s theorem to calculate the work done by the force field $\vec{F}(x, y) = (x^2 - y^3, x^3 + y^2)$ moving a particle around the unit circle $x^2 + y^2 = 1$ in the clockwise direction.
8. **(10 points)** Let $f(x, y, z) = x^2y + y^2z + xyz$.

   a. Calculate $\nabla f$ and $\text{curl}(\nabla f)$.

   b. Calculate the line integral: \[\int_C \nabla f \cdot d\vec{r}\] where $C$ is the top half of the ellipse $3x^2 + 7y^2 = 12$ traversed from $(-2, 0)$ to $(2, 0)$. 
9. **(10 points)** Let \( f(x, y) = x^4 + y^2 - 8x^2 - 6y + 16 \). Find and classify all critical points of \( f(x, y) \) as local max, local min or saddle points.
10. (10 points) Suppose \( w = x^2 e^{yz} \) and \((x, y, z) = (st + u^2, s^2 ut, s + tu)\). Find \( \frac{\partial w}{\partial u} \) at the point where \( s = 1, t = 2, u = 3 \).

11. (10 points) Find the equation of the plane containing the points \((1, 2, 3), (-2, 1, 1), (-3, 1, 2)\).
12. *(10 points)* Let $D$ be the region enclosed by the curves $y = 0$, $y = x^2$ and $x = 1$. Find the average value of $f(x, y) = x \sin y$ over the region $D$. 
13. **(5 points)** Consider a particle with position at time $t$ given by $\vec{r}(t) = (1 + t, t^3, t^2)$. What is the speed of the particle at when $t = 3$?

14. **(10 points)** Let $R$ be the rectangle $[-3, 3] \times [0, 3]$. Estimate $\int \int_R xy^2$ using a Riemann sum with $m = n = 3$ and the upper right corner of each rectangle as your sample point.