

Name:

SOLUTIONS

Math 2950- Midterm Exam #2 - October 29, 2004

1. (15 points) Find $\partial z / \partial s$ at the point where $s = 1$ and $t = 2$.

$$z = 2xy + x^2, \quad x = s + t^2, \quad y = st$$

$$\begin{aligned} \frac{dz}{ds} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = (2y + 2x) \cdot 1 + 2xt \\ & \quad s=1 \quad t=2 \quad x=5 \quad y=2 \\ &= 14 + 20 = \boxed{34} \end{aligned}$$

2. (15 points) Let $f(x, y) = 4x^2 - xy + y^2$. Find the critical point(s) of $f(x, y)$. Use the second derivative test to classify each as a local max, local min or saddle.

$$\nabla f = (8x - y, -x + 2y)$$

Set $\nabla f = (0, 0)$ gives $x=0, y=0$ as only C.P.

$$D = \begin{vmatrix} 8 & -1 \\ -1 & 2 \end{vmatrix} = 15 > 0$$

$$f_{xx} = 8 > 0$$

local min

3. (10 points) Your grade g on this exam depends on the average amount of time t that you study each day and your attendance a since the last exam. Values of the function $g = f(a, t)$ are given in the following table where t is given in hours and a is given in days.

$a \backslash t$	1	2	3	4	5
20	50	55	61	68	76
21	55	61	68	76	84
22	60	64	72	77	87
23	63	67	76	81	89
24	66	70	82	86	95

Estimate the values of $f_a(22, 3)$ and $f_t(23, 4)$.

$$\begin{array}{cc} 15 & 55 \\ 4 & 6.5 \end{array}$$

4. (15 points) Write the equation of the tangent plane to the surface $x^2 - 2yz - z^2 = 1$ at the point $(1, -1, 2)$.

$$\begin{aligned} \nabla f &= (2x, -2z, -2y-2z) \\ \text{at } (1, -1, 2) &= (2, -4, -2) \end{aligned}$$

$$(2, -4, -2) \cdot (x-1, y+1, z-2) = 0$$

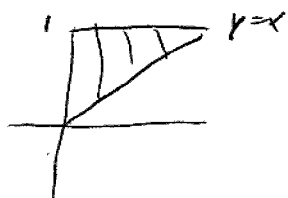
5. (10 points) Find the total differential dz :

$$z = y \cos(xy)$$

$$dz = -y^2 \cos(xy) dx + (\cos(xy) - xy \sin(xy)) dy$$

7. (10 points) Evaluate the double integral by first reversing the order of integration

$$\int_0^1 \int_x^1 \cos(y^2) dy dx.$$



$$\begin{aligned} & \int_0^1 \int_0^y \cos(y^2) dx dy \\ &= \int_0^1 y \cos(y^2) dy \\ &= \frac{1}{2} \sin(y^2) \Big|_0^1 \\ &= \frac{1}{2} \sin 1 \end{aligned}$$

8. (10 points) Compute $\int_D f(x,y) dA$ where $f(x,y) = 1/\sqrt{x^2+y^2}$ and D is the first quadrant region lying between the circles $x^2+y^2=4$ and $x^2+y^2=9$.

In polar we get $0 \leq \theta \leq 2\pi$ $2 \leq r \leq 3$

$$\int_0^{2\pi} \int_2^3 \frac{1}{r} r dr d\theta = \int_0^{2\pi} \int_2^3 dr d\theta$$

$$= 2\pi$$