

1. Let $z = x^2y^3$, $x = t + s \cos(t)$, $y = st^2$. Find $\frac{\partial z}{\partial s}$.

$$\frac{dz}{ds} = \frac{dz}{dt} \frac{dx}{ds} + \frac{dz}{dy} \frac{dy}{ds} = \boxed{2xy^3 \cos t + 3x^2y^2 t^2}$$

OK to sub in for x, y if you want

2. Use the equation $\frac{dy}{dx} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$:

$$\sqrt{xy} = x + x^2y^2$$

$$F = x + x^2y^2 - \sqrt{xy} \quad F_x = 1 + 2xy^2 - \frac{y}{2\sqrt{xy}}$$

$$F_y = 2x^2y - \frac{x}{2\sqrt{xy}}$$

$$\frac{dy}{dx} = -\frac{1 + 2xy^2 - \frac{y}{2\sqrt{xy}}}{2x^2y - \frac{x}{2\sqrt{xy}}}$$

Name:

SOLUTIONS

1. Let $f(x, y) = y^2/x$. Find the maximum rate of change of f at the point $(2, 4)$ and find the direction in which it occurs.

$$\nabla f = \left(-\frac{y^2}{x^2}, \frac{2y}{x} \right), \quad \nabla f(2, 4) = \boxed{(-4, 4)} \leftarrow \text{direction}$$

$$\text{max rate} = \sqrt{4^2 + 4^2} = \boxed{\sqrt{32} = 4\sqrt{2}}$$

2. Let $f(x, y, z) = x^2y + yz + xz$. Find ∇f .

$$\nabla f = (2xy + z, x^2 + z, y + x)$$