

Name: SOLUTIONS

Quiz #7 - March 17, 2009

1. Let $f(x) = \frac{3}{64}x\sqrt{16-x^2}$ for $0 \leq x \leq 4$ and $f(x) = 0$ for all other values of x . Verify that $f(x)$ is a probability density function and find $P(X < 2)$.

$$\int_{-\infty}^{\infty} f(x) dx = \int_0^4 \frac{3}{64} x \sqrt{16-x^2} dx \quad u = 16-x^2 \quad du = -2x dx$$

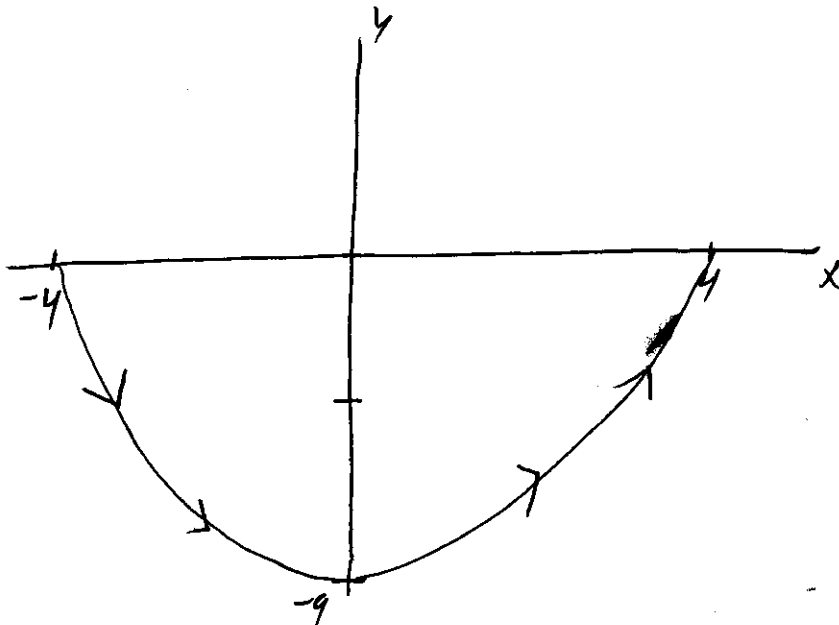
$$= \int \frac{-3}{128} \sqrt{u} du = -\frac{1}{64} (16-x^2)^{3/2} \Big|_0^4$$

$$= -\frac{1}{64} (0 - 64) = 1 \quad \text{SO P.D.F. } \checkmark$$

$$P(X < 2) = \int_0^2 f(x) dx = -\frac{1}{64} (16-x^2)^{3/2} \Big|_0^2$$

$$= -\frac{1}{64} (12^{3/2} - 64)$$

2. Let $x = 4 \cos t$, $y = 9 \sin t$ for $\pi \leq t \leq 2\pi$. Sketch the curve neatly (axes clearly labeled, etc.), indicating with an arrow the direction in which the curve is traced as the parameter increases.



Name:

SOLUTIONS

Quiz #7 - March 19, 2009

1. Let $x = 10 - t^2$, $y = t^3 - 12t$. Find the points on the curve where the tangent line is horizontal or vertical.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 12}{-2t}$$

Horizontal: $3t^2 - 12 = 0$ $t = \pm 2 \rightarrow \boxed{(6, -16), (6, 16)}$

Vertical: $-2t = 0$ $t = 0 \rightarrow \boxed{(10, 0)}$

2. Let $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$. Find the exact length of the curve. Hint for integral: $\sqrt{t^2} = t$ for $t \geq 0$.

$$L = \int_0^1 \sqrt{x'(t)^2 + y'(t)^2} dt = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2} dt$$

$$= \int_0^1 \sqrt{36t^2 + 36t^4} dt$$

$$= \int_0^1 6t \sqrt{1+t^2} dt$$

$$u = 1+t^2 \quad du = 2t dt$$
$$= \int_1^2 3u^{1/2} du = 2u^{3/2} \Big|_1^2$$

$$= \boxed{2(2^{3/2} - 1)}$$