Lecture 8 Integration using Tables

Review u-substitution: Goal is to put an integral into form on our "list".

Ex $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Proof $x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta \rightarrow \int \frac{a\sec^2\theta\,d\theta}{a^2\sec^2\theta} = \int \frac{1}{a}\,d\theta = \frac{\theta}{a}$

But $\theta = \tan^{-1}\left(\frac{x}{a}\right)$ //

Work is done, why repeat.

Ex $\int \frac{e^x}{e^{2x}+9}\,dx$ $u = e^x$ $du = e^x\,dx$

$$= \int \frac{du}{u^2+9} = \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{e^x}{3}\right) + C.$$

Tables are just lists of formulas like $\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$.
You need to recognize which formula to use, keeping in mind that a substitution is usually required to make your integral match the list.

Ex $\int_2^3 \frac{1}{x^2\sqrt{4x^2-7}}$ # 45: $\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$

$u = 2x$ $a = \sqrt{7}$

$$= 4\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{4\sqrt{u^2-a^2}}{a^2u} = \frac{4\sqrt{4x^2-7}}{14x}\Big|_2^3 = \frac{4\sqrt{29}}{42} - \frac{4\sqrt{9}}{28}$$

Ex $\int \frac{\tan^3(1/z)}{z^2}$ #69: $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

$u = 1/z \quad du = -1/z^2\,dz$

$= -\frac{1}{2}\tan^2(1/z) - \ln|\cos(1/z)| + C$

Ex $\int x\sqrt{3x^2+x+1}\,dx$ Formulas 21–29 = Forms involving $\sqrt{a^2+u^2}$

$3x^2+x+1 = (\sqrt{3}x + \frac{1}{2\sqrt{3}})(\sqrt{3}x + \frac{1}{2\sqrt{3}}) + 11/12$

$= \int x\sqrt{(\sqrt{3}x+\frac{1}{2\sqrt{3}})^2 + 11/12}$

$u = \sqrt{3}x + 1/2\sqrt{3} \quad du = \sqrt{3}\,dx$

$x = \sqrt{3}u - 1/2$

Formula 4

$= \int (\sqrt{3}u - 1/2)\sqrt{u^2 + 11/12}\,du$

$= \int \sqrt{3}u\sqrt{u^2+11/12}\,du - 1/2\sqrt{u^2+11/12}\,du$

$v = u^2 + 11/12$

$dv = 2u\,du$

$a = \sqrt{11/12}$

#21 $\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln|u + \sqrt{a^2+u^2}|$

$= \frac{\sqrt{3}}{3}(u^2+11/12)^{3/2} - \frac{1}{2}\left(\frac{u}{2}\sqrt{u^2+11/12} + \frac{11}{24}\ln|u + \sqrt{u^2+11/12}|\right)$

$= \frac{\sqrt{3}}{3}(3x^2+x+1)^{3/2} - \frac{\sqrt{3}x + 1/2\sqrt{3}}{4}\sqrt{3x^2+x+1} - \frac{11}{48}\ln\left|\sqrt{3}x + \frac{1}{2\sqrt{3}} + \sqrt{3x^2+x+1}\right|$

Computer Algebra

MAPLE $\int x^2\,dx \to$ `int(x^2, x);`

$\int 3\sin(x^2)\,dx \to$ `int(3*sin(x^2), x);`

$\int_1^3 x^2+1\,dx \to$ `int(x^2+1, x=1..3);`

Numerical approx

`evalf(int(exp(x^2), x=1..3));` 1443082472

$\int_1^3 e^{x^2}\,dx$

Rmk

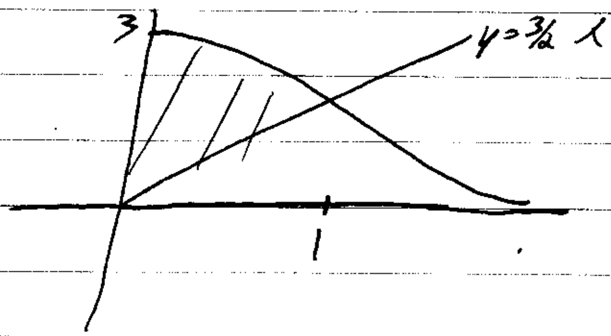
There is no nice formula for $\int e^{x^2}dx$ but

FTOC says $g(t) = \int_1^t e^{x^2}dx$ is a nice continuous function with $g'(t) = e^{t^2}$.

Review

CHPT 6 Area between Curves

Ex Find area between $y = \frac{3}{x^2+1}$ and $y = \frac{3}{2}x$ with $x \geq 0$



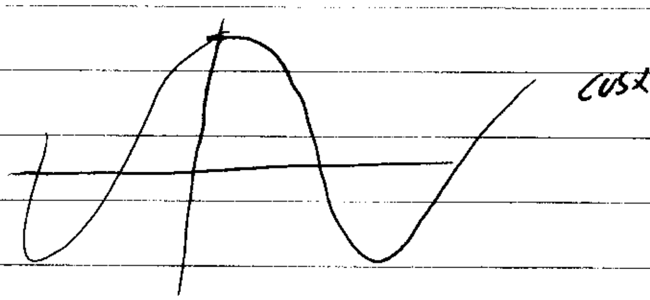
$\frac{3}{2}x = \frac{3}{x^2+1}$

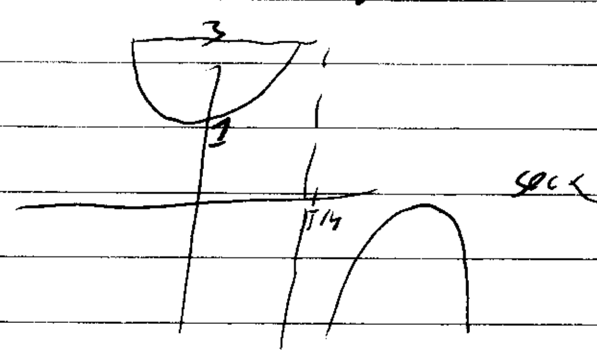
$x = 1$

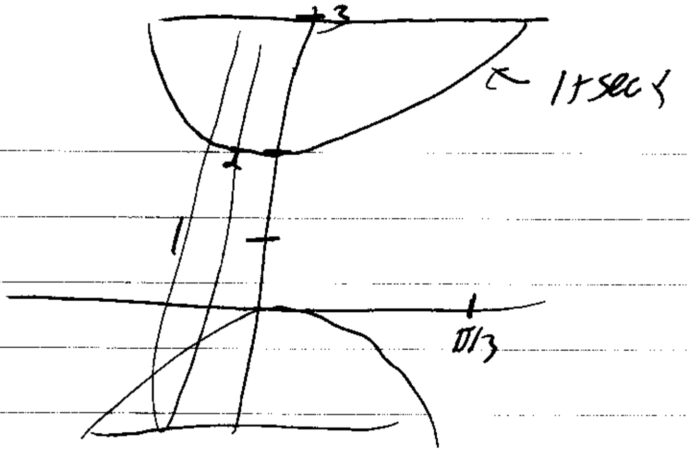
$$\int_0^1 \frac{3}{x^2+1} - \frac{3}{2}x\,dx = 3\tan^{-1}x - \frac{3}{4}x^2 \Big|_0^1$$
$$= \left(3\frac{\pi}{4} - \frac{3}{4}\right) - (0)$$
$$= \frac{3}{4}(\pi - 1)$$

Ex Volumes $V = \int A(x)\,dx$

Find volume rotating $y = 1 + \sec x$, $y = 3$ about $y = 1$.







$1 + \sec x = 3$

$\sec x = 2$

$\cos x = 1/2 \quad x = \pi/3, -\pi/3$

$$V = \int_{-\pi/3}^{\pi/3} \pi(2^2) - \pi(\sec^2 x)\,dx$$

$$= \int_{-\pi/3}^{\pi/3} 4\pi - \pi\sec^2 x\,dx$$

$$= 4\pi x - \pi\tan x \Big|_{-\pi/3}^{\pi/3}$$

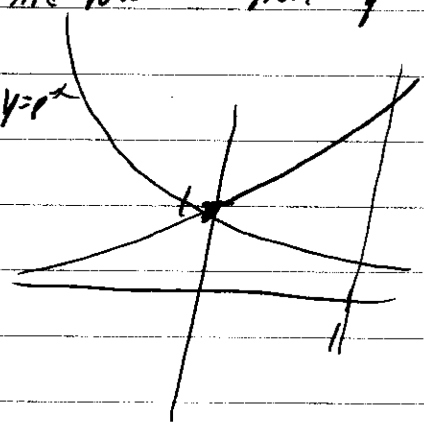
$$= \left(\frac{4}{3}\pi^2 - \pi\frac{\sqrt{3}}{2}\right) - \left(-\frac{4}{3}\pi^2 + \pi\frac{\sqrt{3}}{2}\right)$$

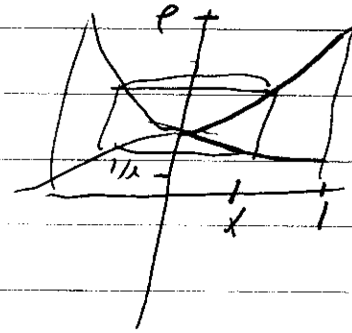
$$= \frac{8}{3}\pi^2 - \pi\sqrt{3}$$

<u>Shells</u>

1. Sketch, determine which way to integrate
2. Draw arbitrary shell, find h, r in terms of variable from #1.
3. $V = \int 2\pi r h \Delta V$

Ex Find volume region $y=e^x$, $y=e^{-x}$, $x=1$ about y axis





Shell: $R = x$ $H = e^x - e^{-x}$

$$V = \int_0^1 2\pi x (e^x - e^{-x})\,dx$$

IBP!