

Lecture 1

- go over syllabus, labs, Webassign, calculators

Antiderivatives

Recall: $\int f(x) dx = F(x)$ means $F'(x) = f(x)$ indefinite integral.

$$\text{Ex } \int x^2 dx = \frac{x^3}{3} + C$$

$$\text{Definite Integral: } \int_1^3 x^2 dx = \frac{x^3}{3} \Big|_1^3 = 9 - \frac{1}{3} = \frac{26}{3}$$

Differentiation Rules and Formulas give corresponding rules for integrals:

Formulas

$$1. \int k dx = kx + C$$

$$2. \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

$$3. \int \frac{1}{u} du = \ln|u| + C$$

$$4. \int a^u du = \frac{a^u}{\ln a} + C$$

$$5. \int e^u du = e^u + C$$

$$6. \int \cos u du = \sin u + C$$

$$7. \int \sin u du = -\cos u + C$$

$$8. \int \sec^2 u du = \tan u + C$$

$$9. \int \csc^2 u du = -\cot u + C$$

$$10. \int \sec u \tan u du = \sec u + C$$

$$11. \int \csc u \cot u du = -\csc u + C$$

$$12. \int \frac{1}{1+u^2} du = \tan^{-1} u + C$$

$$13. \int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + C$$

$$14. \int \sinh u du = \cosh u + C$$

$$\int \cosh u du = \sinh u + C$$

Rules

1. $\int c \cdot f(u) du = c \int f(u) du$ 2. $\int f(u) \pm g(u) du = \int f(u) du \pm \int g(u) du$

Goal! Uses these formulas to compute many more antiderivatives.

Step 1 Learn 1-14, 1-2 cold!

Step 2 Recall if $u = g(x)$ then $du = g'(x) dx$ (differential)

Also $\frac{d}{dx}(f(u)) = f'(g(x)) \cdot g'(x) dx$

Example

1. $\int 2t \cos(t^2) dt$ RMK $2t \cos(t^2)$ not on list!

Let $u = t^2$ $du = 2t dt$

$$\int 2t \cos(t^2) dt = \int \cos u du = \sin u + C = \sin(t^2) + C$$

2. $\int x e^{x^2+1} dx$ $u = x^2+1$ $du = 2x dx$
So $x dx = \frac{1}{2} du$

$$\begin{aligned} \int x e^{x^2+1} dx &= \int \frac{1}{2} e^u du = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C \\ &= \frac{1}{2} e^{x^2+1} + C \end{aligned}$$

Procedure for "u-substitution"

1. Choose u - goal is to simplify \int , get to known list
2. Compute du
3. Replace all "x" terms with "u" terms and integrate
4. Replace "u"s with "x"s

More examples

$$\begin{aligned}
 3 \quad \int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \quad u = \cos x \quad du = -\sin x \, dx \\
 &= \int -\frac{1}{u} \, du = -\ln|u| + C = -\ln|\cos x| + C \\
 &= \ln\left|\frac{1}{\cos x}\right| + C \\
 &= \ln|\sec x| + C
 \end{aligned}$$

$$\boxed{\int \tan x \, dx = \ln|\sec x| + C}$$

Also

$$\int \cot x \, dx = \ln|\sin x| + C = -\ln|\csc x| + C$$

$$\begin{aligned}
 4 \quad \int \sqrt{1+x^2} \, x^5 \, dx \\
 u = 1+x^2 \quad du = 2x \, dx \quad \text{so } x^2 = u-1
 \end{aligned}$$

$$\begin{array}{ccc}
 \sqrt{1+x^2} & x^4 & x \, dx \\
 \uparrow & \uparrow & \downarrow \\
 \sqrt{u} & (u-1)^2 & \frac{1}{2} du
 \end{array}$$

extra work in step 3!

$$\begin{aligned}
 &= \int \sqrt{u} (u-1)^2 \frac{1}{2} du = \frac{1}{2} \int u^{1/2} (u^2 - 2u + 1) \, du \\
 &= \frac{1}{2} \int u^{5/2} - 2u^{3/2} + u^{1/2} \, du \\
 &= \frac{1}{2} \left(\frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right) + C \\
 &= \frac{1}{2} \left(\frac{2}{7} (1+x^2)^{7/2} - \frac{4}{5} (1+x^2)^{5/2} + \frac{2}{3} (1+x^2)^{3/2} \right) + C
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \int \cos(2x) \, dx \quad u = 2x \quad du = 2 \, dx \\
 \int \frac{1}{2} \cos u \, du = \frac{1}{2} \sin u + C = \frac{1}{2} \sin(2x) + C
 \end{aligned}$$

(4)

u-substitution for definite integrals $\int_a^b f(x) dx$

Method 1 Ignore a, b . Do $\int f(x) dx$ using u -subs, get back in terms of x and plug in b, a .

Ex $\int_1^2 x e^{x^2+1} dx = \frac{1}{2} e^{x^2+1} \Big|_{x=1}^{x=2} = \frac{1}{2} (e^5 - e^2)$

Method 2 Change limits of integration $x=a, x=b$ to $u=--, --$. Skip "step 4," just plug in u limits.

Ex $\int_1^2 x e^{x^2+1} dx$ $u = x^2 + 1 \quad du = 2x dx$
 $x=1 \rightarrow u=2$
 $x=2 \rightarrow u=5$

$$= \int_2^5 \frac{1}{2} e^u du = \frac{1}{2} e^u \Big|_{u=2}^{u=5} = \frac{1}{2} (e^5 - e^2)$$

Prmk Either method is OK, same answer.

Ex $\int_1^2 \frac{e^{1/x}}{x^2} dx$ $u = 1/x \quad du = -1/x^2 dx$
 $x=1 \rightarrow u=1$
 $x=2 \rightarrow u=1/2$

$$\int_1^{1/2} -e^u du = -e^u \Big|_1^{1/2} = -(e^{1/2} - e) = e - \sqrt{e}$$

Application Recall $f(x)$ is odd if $f(-x) = -f(x)$
even if $f(x) = f(-x)$

Thm Suppose f is continuous on $[-a, a]$.

1. f even then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

2. f odd then $\int_{-a}^a f(x) dx = 0$

Proof $\int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$

$u = -x \quad du = -dx$

$\int_a^0 f(-u) du = \int_0^a f(-u) du$

$= \begin{cases} \int_0^a -f(u) du & \text{if odd} \\ \int_0^a f(u) du & \text{if even} \end{cases} //$

Problems

1. ~~$\int \tan x dx$~~ $\int x(x^2-5)^{15} dx$

2. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

3. $\int \frac{e^x}{e^x+1} dx$

4. $\int \frac{x}{1+x^4} dx$

5. $\int_0^1 \frac{e^z+1}{e^z+2} dz$