Recall: Suppose \( F(x) \) is an antiderivative of \( f(x) \), i.e. \( F'(x) = f(x) \). Then

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a)
\]

This inspires new notation:

\[
\int f(x) \, dx = F(x) \\
\text{means } F'(x) = f(x)
\]

\( \int f(x) \, dx \) is called an \textbf{indefinite integral}!

\( \int_{a}^{b} f(x) \, dx \) is a \textbf{definite integral}!

Example:

\[
\int \cos x \, dx = \sin x + C
\]

\[
\int t^3 \, dt = \frac{1}{4} t^4 + C
\]

\[
\int 5 \, dx = 5x + C
\]

\textbf{Warning: } \int_{a}^{b} f(x) \, dx \text{ is a function}

\[
\int_{a}^{b} f(x) \, dx \text{ is a number}
\]
Remark

Our huge list of derivatives now becomes a list of formulas for indefinite integrals.

Ex.

\[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1 \]
\[ \int \sec^2 x \, dx = \tan x + C \]
\[ \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C \]
\[ \int \frac{1}{x} \, dx = \ln |x| + C \quad \text{etc etc...} \]

Problem

Find \[ \int \sqrt[3]{x^3} + \sqrt[5]{x^5} \, dx \]

\[ = \int x^{3/3} + x^{5/5} \, dx = \frac{x^{3/2}}{3/2} + \frac{x^{5/3}}{5/3} + C \]

\[ = \frac{2}{3} x^{3/2} + \frac{3}{5} x^{5/3} + C \]
**Interpretation of FTOC**

We can restate FTOC as \[ \int_a^b F(x) \, dx = F(b) - F(a). \]

* Definite integral of rate of change of \( F(x) \) = Net change in \( F(x) \) from \( a \) to \( b \).

**Example**

\[ \int_a^b v(t) \, dt = \text{total displacement}, \]

i.e., position at time \( b \) minus at time \( a \).

\[ \int_a^b \Delta P \, dt = \text{Actual population change} \]

\[ \int_t^b \text{speed} \, dt = \int_t^b |v(t)| \, dt = \text{total distance traveled} \]

**Problem**

#57 The velocity in m/s of a particle is given by \( v(t) = 3t - 5 \), \( 0 \leq t \leq 3 \).

Find a) displacement and b) total distance travelled.

a. \[ \int_0^3 (3t - 5) \, dt = \frac{3}{2} t^2 - 5t \bigg|_0^3 = \frac{27}{2} - 15 = -3\frac{3}{2} \]

b. \[ \int_0^3 3t - 5 \, dt + \int_0^3 3 - 5 \, dt = \frac{5t}{2} \bigg|_0^3 + \frac{3}{2} \bigg|_0^3 = \frac{15}{2} + \frac{3}{2} = 9 \]

Total distance travelled = \((v1/6)\)
Problem
If \( f(x) \) is the slope of a trail at \( x \) miles from the start, what does \( \frac{5}{3} \int f(x) \, dx \) represent?

Ans: The change in elevation between 3 miles and 5 miles.

Problem
If oil leaks from a tank at 116 gallons/minute, what does \( \int_0^2 116 \, dt \) represent?

Ans: Total leakage in first two hours.