Review

Power Rule: \( \frac{d}{dx} (x^n) = nx^{n-1} \) any \( n \in \mathbb{R} \)

Exponentials: \( \frac{d}{dx} (a^x) = a^x \ln a \), in particular \( \frac{d}{dx} (e^x) = e^x \)

What about \( xe^x \)?

**Chain (Product Rule)** Suppose \( f(x) \) and \( g(x) \) are both differentiable. Then so is \( f(x)g(x) \) and

\[
\frac{d}{dx} (f(x)g(x)) = \frac{d}{dx} f(x) \cdot g(x) + f(x) \frac{d}{dx} g(x),
\]

Alternate Way to write it:

\[
(fg)' = f'g + fg'
\]
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(fg)' = fg' + f'g
\]

**Example**

\( f(x) = xe^x \) \( f'(x) = e^x + xe^x \)

\( f(x) = x \cdot x \) \( f'(x) = 1 - x + x \cdot 1 = 2 \cdot x \)

\( (fg)\)' = (f\(g)\)' = (f\(g)\)'h + f\(g)\h'

\[
= \frac{d}{dx} (f'g + fg')
\]

each gets differentiated once!
Proof:

\[
\frac{d}{dx} f(g(x)) = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}
\]

\[
= \lim_{h \to 0} \frac{f(u+\Delta u) - f(u)}{\Delta u} = f'(u)
\]

\[
= \lim_{h \to 0} \frac{f(u+\Delta u) - f(u)}{\Delta u}
\]

\[
= \lim_{h \to 0} \frac{f(\Delta u)}{\Delta u} + \lim_{h \to 0} g(x) \cdot \frac{f(\Delta u) - f(x)}{\Delta u}
\]

\[
= \lim_{h \to 0} \frac{f(\Delta u) - f(x)}{\Delta u}
\]

F is continuous

\[
= f'(x) g'(x) + g(x) f'(x)
\]

Quotient Rule

\[
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}
\]

A.K.A.

\[
\left( \frac{f}{g} \right)' = \frac{g f' - f g'}{g^2}
\]

"low de hi minus hi de low over low squared"
Ex
1. \( g(x) = \frac{e^x}{1 + x^2} \) Find \( g'(1) \).

2. \( h(x) = \frac{x e^x}{1 + x^2} \) Find \( h'(1) \).

3. Find tangent line to \( y = \frac{x^3}{1-x^2} \) at \((2, -\frac{8}{3})\).

\[ F(x) = \text{Area} \] Estimate \( F'(1) \).
5. Find equations of tangent lines to \( y = \frac{x-1}{x^2} \)

which are \( \parallel \) to \( x-2y=2 \).

6. \( g(x) = xe^x \). Find \( g''(x) \).