

Reduction of collision-induced timing shifts in dispersion-managed quasi-linear systems with periodic-group-delay dispersion compensation

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Periodic-group-delay (PGD) dispersion-compensation modules were recently proposed as mechanisms to alleviate collision-induced timing shifts in dispersion-managed (DM) systems. Frequency and timing shifts in quasi-linear DM systems with PGDs were obtained, and it is shown that significant reductions are achieved when even a small fraction of the total dispersion is compensated for by PGDs. © 2004 Optical Society of America

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Collision-induced timing shifts (TSs), that is, changes in the arrival time of a signal owing to its interaction with signals in neighboring frequency channels, is a major issue in on-off-keying, wavelength-division-multiplexed (WDM) systems. The problem is complex if dispersion management (DM) is employed: because of the periodic change with distance of the sign of chromatic dispersion, pulses in different frequency channels undergo a zigzag motion with respect to each other, which results in interactions that consist of a series of minicollisions. Furthermore, the low value of average dispersion in these systems results in large collision lengths, implying that the interaction unfolds over long distances. Indeed, whereas it is well known that a moderate amount of dispersion management can be beneficial, in extreme cases collisions can result in mutual destruction of the two WDM pulses.¹

Recently the use of periodic-group-delay (PGD) modules was proposed as a means of reducing collision-induced TS.²⁻⁴ PGD dispersion-compensating modules⁵⁻⁷ are inserted inside each dispersion map and serve to compensate for a fraction of the accumulated dispersion, with the remaining fraction of dispersion compensation achieved by dispersion-compensating fibers. The advantage of this approach for DM soliton systems has been demonstrated both with numerical simulations^{2,4} and with direct transmission experiments.³

In this Letter we derive expressions for the value of the collision-induced frequency shifts (FSs) and TSs in quasi-linear return-to-zero systems that employ PGDs. Second, we use both our analytical results and direct numerical simulations (DNSs) to show that the use of PGDs is highly effective in reducing collision-induced TSs in quasi-linear, return-to-zero systems. As with solitons, the main mechanism for reduction in TS is the drastic reduction of the collision length.² We show that there also is a significant direct effect from the PGDs themselves, which provide a restoring force to the TS.

We start from the nonlinear Schrödinger equation written in dimensionless variables, $t = t_{\text{ret}}/t_*$, $z = z_{\text{lab}}/z_*$, $u = \mathcal{E}[g(z)P_*]^{1/2}$, and $D = -k''/k_*''$,

with normalization parameters denoted by asterisks. Here \mathcal{E} is the slowly varying complex optical field envelope and t_{ret} and z_{lab} are, respectively, the retarded time and the propagation distance. Typical normalization values used are $P_* = 1$ mW, $z_* = z_{\text{NL}} = 1/(\gamma P_*) = 400$ km, $t_* = 12$ ps, and $k_*'' = t_*^2/z_* = 0.36$ ps²/km, where $\gamma = 2.5$ (W/km)⁻¹ is the fiber's nonlinear coefficient. We consider a dispersion map consisting of two fiber sections (with dispersion coefficients and lengths $D_1 > 0$, z_1 and $D_2 < 0$, z_2) and an erbium-doped fiber amplifier (EDFA) located after the second fiber. The total map length is $z_a = z_1 + z_2$, and the fraction of the map consisting of the anomalous fiber is $\theta = z_1/z_a$. Here $g(z)$ describes the periodic power variation that is due to loss and amplification, and $D(z)$ is the local group-velocity dispersion, both periodic functions with period z_a . With EDFAs, $g(z) = g_0 \exp(-2\Gamma z)$ for $nz_a < z < (n+1)z_a$, where n is the map number, Γ is a dimensionless loss coefficient, and $g_0 = 2\Gamma z_a/[1 - \exp(-2\Gamma z_a)]$. A measure of the effects of dispersion management is given by the map strength⁸ $s = [(D_1 - \langle D \rangle)z_1 - (D_2 - \langle D \rangle)z_2]/4$, where $\langle D \rangle = (D_1 z_1 + D_2 z_2)/z_a$ is the average dispersion. For a single pulse with normalized frequency Ω_0 , its mean position $\langle t \rangle$ at distance z along the fiber is given by $\langle t \rangle = \Omega_0 \int_0^z D(x) dx = \Omega_0[\langle D \rangle z + C(z)]$, where $C(z) = \int_0^z [D(x) - \langle D \rangle] dx$ is periodic. Suppose that two pulses u_{\pm} with frequencies $\pm\Omega_0$ are initially located at $\mp t_0$; the initial time displacement t_0 is related to the mean collision location z_0 by $t_0 = \Omega_0 \langle D \rangle z_0$. Minicollisions between the pulses then occur when $(z - z_0) \langle D \rangle + C(z) = 0$. We can estimate the collision length to be $L_c = 2s/\langle D \rangle$. Complete collisions are those for which $L_c/2 \leq z_0 \leq L - L_c/2$.

In systems with PGDs, a PGD module is inserted in each dispersion map, providing a local dispersive force with strength H_{PGD} and replacing a fraction f of the dispersion provided by the second fiber. That is, $f = H_{\text{PGD}}/(D_2 z_2' + H_{\text{PGD}})$, where hereafter a prime denotes a quantity in a system with PGDs. We consider this replacement to be made while the average dispersion, the individual dispersions, and the length of the first fiber section are unchanged: $D'_{1,2} = D_{1,2}$,

$z'_1 = z_1$, and $\langle D \rangle' = (D_1 z'_1 + D_2 z'_2 + H_{\text{PGD}})/z'_a = \langle D \rangle$. We then find the new system parameters to be $\theta' = [(D_1 - D_2)(1 - f)\theta - D_2 f]/[D_1(1 - f) - D_2]$, $z'_a = (\theta/\theta')z_a$, and $z'_2 = (1 - \theta')z'_a$. It is useful to define an effective average dispersion: $\langle D \rangle_{\text{eff}} \equiv (D_1 z_1 + D_2 z_2)/z_a$. Using the equivalent definition of map strength $2s = (D_1 - \langle D \rangle)z_1$, we then find the new map strength given by $s' = [(D_1 - \langle D \rangle_{\text{eff}})/(D_1 - \langle D \rangle)]s$. The new collision length is then $L'_c = 2s'/\langle D \rangle_{\text{eff}}$. Propagation of optical pulses in such a system is governed by a perturbed nonlinear Schrödinger equation:

$$iu_z + 1/2 D(z)u_{tt} + g(z)|u|^2 u = iP[u], \quad (1a)$$

$$\hat{P}[u] = \{\exp[iH(\omega)] - 1\} \sum_{m=1}^{N_a} \delta(z - mz'_a) \hat{u}(\omega, z) \quad (1b)$$

(see Ref. 9), where $P[u]$ expresses the action of the PGDs in the time domain, $\hat{P}[u]$ is the Fourier transform of $P[u]$ and $\hat{u}(\omega, z)$ is that of $u(t, z)$, and N_a is the total number of dispersion maps in the transmission line. $H(\omega)$ is the PGD response, which locally approximates a quadratic dispersive profile but is periodic with a period equal to the channel spacing, in our case $2\Omega_0$. Finally, $\delta(z)$ is the Dirac delta function, which encodes the pulse change across a PGD: $\hat{u}(\omega, mz'_a^+) = \exp[iH(\omega)]\hat{u}(\omega, mz'_a^-)$.

Inserting $u = u_+ + u_-$ into Eqs. (1) and neglecting four-wave mixing, one finds the evolution for u_{\pm} as Eqs. (1) with an extra term on the left-hand-side: namely, $2g(z)|u_{\mp}|^2 u_{\pm}$ in the equation for u_{\pm} . The mean time and pulse frequency are defined as usual by $\langle t \rangle = \int t|u_{\pm}|^2 dt/E$ and $\Omega = -i \int u_{\pm}^* (\partial u_{\pm} / \partial t) dt/E$, where $E = \int |u_{\pm}|^2 dt$ is the pulse energy. (All integrals are from $-\infty$ to ∞ unless otherwise indicated.) Using Eqs. (1) and a second-order Taylor approximation of $\hat{P}[u]$ about $\omega = \Omega_0$ with $H(\Omega_0) = H'(\Omega_0) = 0$, we find evolution equations for the mean time and frequency. Looking at u_+ :

$$\frac{\partial \Omega}{\partial z} = \frac{2g(z)}{E} \int |u_+|^2 \frac{\partial |u_-|^2}{\partial t} dt, \quad (2a)$$

$$\frac{\partial \langle t \rangle}{\partial z} = D(z)\Omega(z) + H_{\text{PGD}} \sum_{m=1}^{N_a} \delta(z - mz'_a) (\Omega - \Omega_0), \quad (2b)$$

where $H_{\text{PGD}} = H''(\Omega_0)$. The TSs and FSs are then $\Delta\Omega = \Omega(z) - \Omega_0$ [with $\Omega(0) = \Omega_0$], and $\Delta t(z) = \langle t(z) \rangle - \bar{D}(z)\Omega_0 + t_0$, where $\bar{D}(z) \equiv \int_0^z D(x) dx$ would be the accumulated dispersion in the new system without PGDs. It is also convenient to define the dispersion including PGDs as $D_{\text{PGD}}(z) \equiv D(z) + H_{\text{PGD}} \sum_{m=1}^{N_a} \delta(z - mz'_a)$ and the accumulated dispersion including PGDs as $\bar{D}_{\text{PGD}}(z) \equiv \int_0^z D_{\text{PGD}}(x) dx$. The evolution with distance of a quasi-linear pulse with an initial Gaussian profile $u_{\pm}(t, 0) = (a/\sqrt{2\pi b}) \exp[-(t \pm t_0)^2/2b \pm i\Omega_0 t]$ is then given by¹⁰

$$u_{\pm}(t, z) = \frac{a}{\{2\pi[b + i\bar{D}_{\text{PGD}}(z)]\}^{1/2}} \times \exp\left\{-\frac{[t \pm t_0 \mp \Omega_0 \bar{D}(z)]^2}{2[b + i\bar{D}_{\text{PGD}}(z)]} \pm i\Omega_0 t - \frac{i}{2} \Omega_0^2 \bar{D}(z)\right\}. \quad (3)$$

Using Eq. (3) in Eqs. (2), we find that

$$\Delta\Omega(L) = A\Omega_0 \int_0^L \frac{g(z)\bar{D}_0(z)}{[b^2 + \bar{D}_{\text{PGD}}^2(z)]^{3/2}} \times \exp\left[-2b\Omega_0^2 \frac{\bar{D}_0^2(z)}{b^2 + \bar{D}_{\text{PGD}}^2(z)}\right] dz, \quad (4a)$$

$$\Delta t(L) = \bar{D}_0(L)\Delta\Omega(L) - \Delta t_{\text{res}} + H_{\text{PGD}} \sum_{m=1}^{N_a} \Delta\Omega(mz'_a), \quad (4b)$$

where $\bar{D}_0(z) \equiv \bar{D}(z) - \langle D \rangle_{\text{eff}} z_0$ with $A \equiv 4Eb^{3/2}/\sqrt{2\pi}$, and where residual TS Δt_{res} is

$$\Delta t_{\text{res}}(L) \equiv A\Omega_0 \int_0^L \frac{g(z)\bar{D}_0^2(z)}{[b^2 + \bar{D}_{\text{PGD}}^2(z)]^{3/2}} \times \exp\left[-2b\Omega_0^2 \frac{\bar{D}_0^2(z)}{b^2 + \bar{D}_{\text{PGD}}^2(z)}\right] dz. \quad (4c)$$

Equation (4b) shows there is an additional restoring force that is due to the PGDs. The sum in Eq. (4b) is calculated from Eq. (4a) with L replaced by mz'_a . Note that the theory above permits the calculation of FSs and TSs for complete as well as incomplete collisions. Importantly, the integrals in Eqs. (4a) and (4c) can be efficiently evaluated by numerical quadrature or by asymptotic approximation, by use of the so-called Laplace method.¹¹

The integrals in Eqs. (4a) and (4c) have the generic form $I(\lambda) = \int_0^L F(z) \exp[-\lambda\Phi(z)] dz$, where $\lambda > 0$ is a parameter. For $\lambda \gg 1$ the main contribution to $I(\lambda)$ comes from a neighborhood of a so-called critical point z_c , where $\Phi(z)$ is minimum; other regions contribute exponentially smaller terms. The functions $F(z)$ and $\Phi(z)$ can then be expanded in Taylor series about the critical point, with typically only the first few terms retained.

In our case, $\Phi(z) = \bar{D}_0^2/(b^2 + \bar{D}_{\text{PGD}}^2)$ and $\lambda = 2b\Omega_0^2$ for both Eqs. (4a) and (4c). Critical points z_c correspond to mini-collision locations, which occur where $\bar{D}_0(z_c) = 0$. For Eq. (4a), $F(z) = g\bar{D}_0/(b^2 + \bar{D}_{\text{PGD}}^2)^{3/2}$; for Eq. (4c), $F(z) = g\bar{D}_0^2/(b^2 + \bar{D}_{\text{PGD}}^2)^{3/2}$. We thus have $F(z_c) = \Phi(z_c) = 0$ for both $\Delta\Omega$ and Δt_{res} but $F'(z_c) \neq 0$ for $\Delta\Omega$ and $F'(z_c) = 0$ for Δt_{res} . Then, if z_1, \dots, z_N are the locations of the minicollisions, we find that

$$\Delta\Omega(L) \approx A \sum_{n=1}^N \left[\frac{F'_n J_{1,n}}{b\Omega_0 \Phi''_n} + \frac{3F''_n \Phi''_n J_{2,n} - 2F'_n \Phi'''_n J_{4,n}}{6b^{3/2} \Omega_0^2 (\Phi''_n)^{5/2}} \right], \quad (5a)$$

$$\Delta t_{\text{res}}(L) \approx \frac{A}{b^{3/2} \Omega_0^2} \sum_{n=1}^N \frac{F''_n J_{2,n}}{2(\Phi''_n)^{3/2}}, \quad (5b)$$

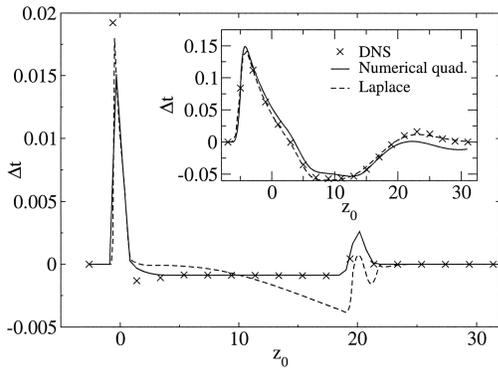


Fig. 1. Total TS versus mean collision location z_0 for $f = 0.1$ from DNSs of Eqs. (1), numerically integrating (quadrature) Eqs. (4) and the asymptotic approximations (Laplace) Eqs. (5).

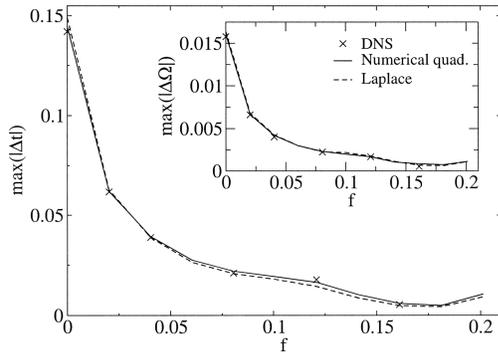


Fig. 2. Maximum (taken of z_0) TS and FS (inset) versus f .

where $F'_n = F'(z_n)$, etc. and $J_{k,n} = \int_{x_n^-}^{x_n^+} x^k \exp(-x^2) dx$, with the limits of integration x_n^\pm depending on the location of the minicollision. The integrals $J_{n,k}$ may be evaluated exactly in terms of well-known functions. Equations (5) show the residual FS and TS are, respectively, $O(1/\Omega_0)$ and $O(1/\Omega_0^2)$. One can further improve the accuracy of the Laplace method by keeping higher-order terms.

We now compare results of DNS of Eqs. (1), results of numerically integrating Eqs. (4) (quadrature), and results of the asymptotic approximations, Eqs. (5) (Laplace). Implicit in our quasi-linear ansatz is that the PGD approximates a quadratic dispersive profile. Thus we used a piecewise, periodic function to model PGDs in Eqs. (1): $H(\omega, z) = (\omega - \Omega_0)^2 H_{\text{PGD}}/2$ for $2n\Omega_0 \leq \omega < 2(n+1)\Omega_0$. Because a PGD is assumed to act locally in space, the pulse was propagated linearly through the spatial cells containing the PGDs, with the jump condition given after Eqs. (1). We thus expect Eqs. (4) to be good approximations when the spectral width of the pulse is small compared with the period of the PGDs, $2\Omega_0$. In the numerical quadrature we evaluated Eqs. (4) by breaking the range of integration $[0, L]$ into subintervals of constant dispersion and using the trapezoidal rule on each subinterval.

We simulated a quasi-linear system with normalized line parameters $L = 20$, $\langle D \rangle = 0.5$, $s = 2.5$, $\theta = 0.5$, $\Gamma = 9.21$, and $z_a = 0.15$ and normalized pulse parameters $a = 1.6$, $b = 2$, and $\Omega_0 = 3$. These values correspond to a system with channel spacing 80 GHz, average dispersion 0.18 ps²/km, total transmission distance 8000 km, loss coefficient 0.2 dB/km, amplifier spacing 40 km without PGDs, and an input pulse with a peak power of 0.6 mW and a FWHM of 28.26 ps. Figure 1 shows the total TS $\Delta t(L)$ versus mean collision location z_0 for a system with ($f = 0.1$) and without ($f = 0$) PGDs. Figure 2 shows the maximum absolute value of the total TS and the residual FS as a function of f , the fraction of compensation performed by the PGDs; the maximum is taken over mean collision location z_0 . In addition to the excellent agreement between analytical formulas and DNSs, these figures clearly show the large reduction in the collision-induced TSs that arises from use of PGDs. Note that the reduction of the TS corresponds to a sharp decrease in the collision length: in Fig. 1, $L'_c = 1.24$ for $f = 0.1$, compared with $L_c = 10$ for $f = 0$ (no PGDs). Note also that $L'_c \ll L$ when f is not too small, which means that, with PGDs, almost all collisions are complete, unlike the case without PGDs. Finally, Fig. 1 shows that, with PGDs, the effect of complete collisions is almost negligible (unlike the case without PGDs), and the only noticeable TSs arise from the few incomplete collisions, which correspond to collision centers z_0 located near the beginning ($z_0 = 0$) or the end ($z_0 = 20$) of the transmission line.

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References

1. M. Ablowitz, G. Biondini, and E. Olson, *J. Opt. Soc. Am. B* **15**, 577 (2001).
2. X. Wei, X. Liu, C. Xie, and L. F. Mollenauer, *Opt. Lett.* **28**, 983 (2003).
3. L. F. Mollenauer, A. Grant, and X. Liu, *Opt. Lett.* **28**, 2043 (2003).
4. X. Liu, W. Wei, L. F. Mollenauer, C. J. McKinstrie, and C. Xie, *Opt. Lett.* **28**, 1412 (2003).
5. M. Shirasaki, *IEEE Photon. Technol. Lett.* **9**, 1598 (1997).
6. C. K. Madsen and G. Lenz, *IEEE Photon. Technol. Lett.* **10**, 994 (1998).
7. X. Shu, K. Sugden, and K. Byron, *Opt. Lett.* **28**, 881 (2003).
8. M. J. Ablowitz and G. Biondini, *Opt. Lett.* **23**, 1668 (1998).
9. M. J. Ablowitz, G. Biondini, S. Chakravarty, and R. Horne, *Opt. Commun.* **172**, 211 (1999).
10. M. J. Ablowitz, T. Hirooka, and G. Biondini, *Opt. Lett.* **26**, 459 (2001).
11. M. J. Ablowitz, A. Docherty, and T. Hirooka, *Opt. Lett.* **28**, 1191 (2003).