Quasi-linear optical pulses in strongly dispersion-managed transmission systems

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A unified analytical description of the evolution of quasi-linear optical pulses and solitons in strongly dispersion-managed transmission systems is developed. Asymptotic analysis of the nonlocal equation that describes the averaged dynamics of a dispersion-managed system shows that the nonlinearity decreases for large map strength s, as $O(\log s/s)$. The spectral intensity is found to be an invariant of the propagation, which allows the phase shift to be computed. These findings provide a clear description of pulse propagation in the quasi-linear regime, which is characterized by much lower energies than those required for stable dispersion-managed soliton transmission with the same dispersion map.

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In recent years, considerable research has been devoted to the study of fiber-optic communication systems employing strong periodic dispersion management for both quasi-linear and soliton transmission. In quasi-linear (low-power) systems, strong dispersion management is used to compensate for fiber dispersion, manage fiber nonlinearity, and suppress interchannel cross talk.[1,2] In soliton systems, dispersion management provides increased robustness to the amplifier noise and the reduction of timing jitter arising from nonlinear interactions between signals (cf. Ref. 3). In strongly dispersion-managed (DM) systems, the nonlinear Schrödinger (NLS) equation that describes the pulse dynamics in optical fibers is perturbed by a large and rapid periodic variation of the dispersion. As a result, the dynamics is asymptotically governed by a nonlocal equation [Eq. (2), below] that is obtained by employment of an appropriate multiscale expansion of the perturbed NLS equation.[4] We refer to this equation as the dispersion-managed nonlinear Schrödinger (DMNLS) equation (see also Ref. 5). DM solitons are solutions of the DMNLS equation and their evolution and interaction properties have been studied in Ref. 6.

In this Letter we study the evolution of optical pulses in the quasi-linear regime based on the DMNLS equation. By employing an asymptotic expansion of the nonlinear term of the DMNLS equation, we quantify the significant reduction of the effective nonlinearity for large map strength. In this regime the nonlinearity yields a phase shift in the frequency domain, preserving the spectral intensity of the pulse during propagation. The analysis shows how strong dispersion management also effectively manages nonlinearity.

We start the analysis from the perturbed NLS equation

$$i\,\frac{\partial u}{\partial z} + \frac{d(z)}{2}\,\frac{\partial^2 u}{\partial t^2} + g(z)\,|u|^2 u = 0\,, \qquad (1)$$

where all quantities are expressed in dimensionless units. The functions $d(z)$ and $g(z)$ describe the local group-velocity dispersion (GVD) of the fiber and the variation of power owing to loss and lumped amplification, respectively, which are both periodic, with period $z_a \ll 1$. As is standard, $g(z)$ is given by $g(z) = g_0 \exp[-2\Gamma(z - nz_a)]$, $nz_a \le z < (n+1)z_a$, with $g_0 = 2\Gamma z_a/[1 - \exp(-2\Gamma z_a)]$, where Γ is the dimensionless loss coefficient.

To model strong dispersion management, we write $d(z)$ as $d(z) = \langle d\rangle + (1/z_a)\Delta(z/z_a)$, where the constant $\langle d\rangle$ represents the path-average dispersion and $\Delta(z/z_a)$ describes the rapid variation of GVD with zero average. An appropriate multiscale expansion of the perturbed NLS equation yields the following nonlocal (DMNLS) equation,[4] represented in the frequency domain for convenience:

$$i\,\frac{\partial \hat{U}}{\partial z} - \frac{\langle d\rangle}{2}\,\omega^2 \hat{U} + I[\hat{U}(z,\omega)] = 0\,, \qquad (2a)$$

$$I[\hat{U}(z,\omega)] = \iint_{-\infty}^{+\infty} \mathrm{d}\omega_1 \mathrm{d}\omega_2 \hat{U}(z,\omega+\omega_1)\hat{U}(z,\omega+\omega_2) \times \hat{U}^*(z,\omega+\omega_1+\omega_2) r(\omega_1\omega_2)\,, \qquad (2b)$$

where $\hat{u}(z,\omega) \sim \hat{U}(z,\omega)\exp[-iC(z/z_a)\omega^2/2]$ and $r(x) = \int_0^1 \mathrm{d}\zeta\, g(\zeta)\exp[iC(\zeta)x]/(2\pi)^2$, with $\mathrm{d}C(\zeta)/\mathrm{d}\zeta = \Delta(\zeta)$ and $\hat{u}(z,\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, u(z,t)\exp(i\omega t)$. Hereafter we consider a two-step dispersion map composed of fiber segments of equal length with positive and negative constant values of GVD, $\Delta(z) = \pm\Delta$, respectively. In this profile the map strength (depth) is given by $s = (d - \langle d\rangle)z_a/4 = \Delta/4$ (note that the map strength is usually defined in terms of dimensional quantities[7]; cf. Ref. 6). For a lossless system [i.e., when $g(z) = 1$] the kernel $r(x)$ is (in the lossy case; see Ref. 6)

$$r(x;s) = \frac{1}{(2\pi)^2}\frac{\sin sx}{sx}\,. \tag{3}$$

It is clear from Eq. (3) that $r \to 0$ as $s \to \infty$ (see also Refs. 8 and 9) but the precise estimate requires some detailed analysis.

To analyze the behavior of quasi-linear optical pulses using Eqs. (2) and (3), we assume that $\hat{U}(z,\omega)$ depends only weakly on s. (This assumption is not valid for DM solitons, owing to the large dependence of pulse energy on s.[7]) Asymptotic evaluation of Eqs. (2b) with Eq. (3) then leads to

$$I[\hat{U}(z,\omega)] = \Psi(|\hat{U}(z,\omega)|^2)\hat{U}(z,\omega) + O(1/s^2)\,, \tag{4a}$$

$$\Psi[|\hat{U}(z,\omega)|^2] = \frac{1}{2\pi s}\Bigg[(\log s - \gamma)\,|\hat{U}(z,\omega)|^2 - \int_{-\infty}^{\infty} \mathrm{d}\omega' f(\omega'-\omega)\,|\hat{U}(z,\omega')|^2\Bigg], \tag{4b}$$

where $\gamma = 0.57722$ is Euler's constant[10] and $f(\omega) = (1/\pi)\int_{-\infty}^{\infty}\mathrm{d}t\, \log|t|\exp(i\omega t)$. This result is obtained either by use of the method of stationary phase in Eq. (2b) with Eq. (3) or by writing the DMNLS equation in the time domain with the corresponding integration kernel,[4] $R(tt') = \mathrm{ci}(|tt'/s|)/(2\pi|s|)$, where $\mathrm{ci}(x) = \int_1^{\infty}\mathrm{d}y\, \cos(xy)/y$. [As $x \to 0$, $\mathrm{ci}(x) \sim -\gamma - \ln x + O(x)$ (Ref. 10)]. Neglecting $O(1/s^2)$ terms, we find that the spectral intensity $|\hat{U}(z,\omega)|^2$ is preserved during pulse propagation with Eq. (2a). Correspondingly, the solution of Eq. (2a) is

$$\hat{U}(z,\omega) = \hat{U}(0,\omega)\exp\{i\langle d\rangle\omega^2 z/2 + i\Psi[|\hat{U}(0,\omega)|^2]z\}\,. \tag{5}$$

The linear phase shift can be corrected by pretransmission or posttransmission compensation. After this linear phase is removed, or if the system has a small value of path-average dispersion ($\langle d\rangle \ll 1$), the averaged dynamics of the quasi-linear pulse transmission is characterized only by the nonlinear phase shift $\phi_{\mathrm{NL}}(z,\omega) = \Psi[|\hat{U}(0,\omega)|^2]z$. We also note that the large value of s effectively reduces the nonlinearity by $O(\log s/s)$, or, equivalently, the nonlinear distance is increased by $2\pi s/\log s$. Therefore a pulse in a strongly DM system will be able to propagate for much longer distances before being distorted by the nonlinearity, as opposed to a pulse with the same energy in a system with constant dispersion. As a result, strong dispersion management allows stationary pulse propagation in the quasi-linear regime, with energies comparable to those of classical NLS solitons but at the same time much lower than the energies required for formation of a stable DM soliton at $\langle d\rangle \sim 0$ for the same value of s, owing to the energy enhancement of DM solitons.[7] These results provide a clear analytical description of quasi-linear return-to-zero pulses with large map strength, as employed in recent experiments and numerical simulations.[11,12]

The DMNLS equation is applied with strong dispersion management independently of the transmission format. Hence this equation describes the dynamics of both DM solitons and quasi-linear pulses within the approximation of the DMNLS equation. In many situations, DM solitons can be approximated by $u_{\mathrm{DMS}}(z,t) \sim \alpha[2\pi\xi(z)]^{-1/2}\exp[-t^2/2\xi(z)]\exp(i\lambda^2 z/2)$, where $\xi(z) = \beta + iC(z)$ and λ is directly related to the pulse energy.[6] Indeed, quasi-linear modes with initial Gaussian shape, ignoring the nonlinear phase shift ϕ_{NL} with $\langle d\rangle \ll 1$, propagate in a similar fashion to DM solitons but with $\lambda = 0$. This shows that the two transmission formats are closely related, which is in consistent with recent numerical observations in Ref. 13.

To test our model, we compared the analytical results with direct numerical simulations of Eq. (1). In the following, we take the incident pulse to be a Gaussian $u(0,t) = (1/\sqrt{\beta})\exp(-t^2/\beta)$. The nonlinear phase shift at $\omega = 0$ is then given by $\phi_{\mathrm{NL}}(z,0) = (1/s)\log(s/\beta)z$. We also restrict ourselves to the particular case of $\langle d\rangle = 0$, although we reiterate that, even in the presence of $\langle d\rangle$, similar results can be obtained by employment of postcompensation of the cumulative GVD. In Fig. 1 we plot the shape of the quasi-linear pulse for $s = 100$ and $\Gamma = 0$ after a propagation of $z = 20$, as well as the initial profile with $\beta = 1.0$. As predicted from the model, the spectral intensity is an invariant of the propagation, whereas small pulse broadening induced by the nonlinearity is observed in temporal domain as a result of the acquired nonlinear chirp $\partial\phi_{\mathrm{NL}}/\partial\omega$ in the frequency domain (which is similar to the self-phase modulation in the time domain). In simulations including loss, we observe only a small spectral narrowing of the pulse. Figure 2 shows a plot of the phase shift $\phi_{\mathrm{NL}}(z,\omega = 0)$ at $z = 1$ as a function of s, in both the lossless ($\Gamma = 0$) and lossy ($\Gamma = 10$) cases. Remarkable agreement between the analytical and the numerical results can be seen in Fig. 2, which confirms the validity of the asymptotic expansion of Eq. (4b). In addition, although the asymptotic analysis above is restricted to the lossless case, this result indicates that the lossy case will follow similar lines. Note that stationary transmission is possible in the quasi-linear regime with energy $E_{\mathrm{RZ}} = 1.77$, which is compatible with

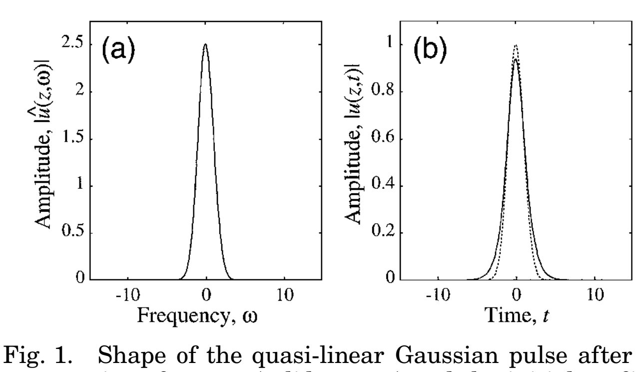

Fig. 1. Shape of the quasi-linear Gaussian pulse after a propagation of $z = 20$ (solid curves) and the initial profile (dotted curves) for $s = 100$, $\langle d\rangle = 0$, and $\Gamma = 0$: (a) frequency domain, (b) time domain. In (a), the dotted curve is indistinguishable from the solid curve.

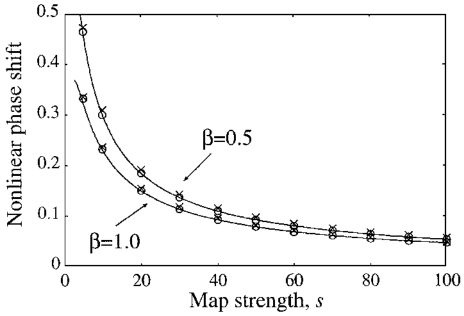

Fig. 2. Nonlinear phase shift $\phi_{\mathrm{NL}}(\omega = 0)$ of the quasi-linear Gaussian pulse with $\beta = 0.5, 1.0$. The solid curve is the analytical result (see text), and the circles and crosses are the numerical results obtained from the direct simulation of Eq. (1) with $\Gamma = 0$ and $\Gamma = 10$, respectively.

the energy required for DM soliton propagation with moderate map strength[14] ($E_{\mathrm{DMS}} = 1.5$ for $s = 8.5$ and $\langle d \rangle = 1.2$). However, the DM soliton at $s = 100$ and $\langle d \rangle = 0$ has energy $E_{\mathrm{DMS}} \gg E_{\mathrm{RZ}}$.

The nonlinear chirp can induce significant pulse distortions with smaller values of s. However, we can use the quadratic approximation of ϕ_{NL} to compensate partially for the nonlinear broadening by expanding ϕ_{NL} in Taylor series with respect to ω: $\phi_{\mathrm{NL}}(z, \omega) = \phi_{\mathrm{NL}}(z, 0) + \phi''_{\mathrm{NL}}(z, 0)\omega^2/2 + \ldots$. In particular, for the Gaussian pulses considered here we have $\phi''_{\mathrm{NL}}(z, 0) = (2\beta/s)[\log(s/\beta) - 2]z$. We numerically compared the rms pulse width before and after compensation, using $s = 10$. We found an initial pulse with a rms of 0.707 broadens to 1.254 at $z = 20$ without compensation. But with compensation rms is returned to 0.771, which is a significant improvement.

In conclusion, we have presented a unified analytical model for studying the dynamics of quasi-linear optical pulses and solitons in strongly dispersion-managed systems. The evolution of the quasi-linear mode is governed by the DMNLS equation, where the nonlinearity, mitigated by $O(\log s/s)$, is responsible for a phase shift in the frequency domain, resulting in some pulse broadening in temporal domain. Thus via the DMNLS equation we establish a striking difference between quasi-linear modes where nonlinearity is suppressed and DM solitons where nonlinearity balances dispersion. Finally, we note that strongly DM systems suffer from nonlinear intrachannel interactions,[15,16] which may need the use of dispersion management with moderate map strengths.

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References

1. C. Kurtzke, IEEE Photon. Technol. Lett. **5,** 1250 (1993).
2. R. W. Tkach, A. R. Chraplyvy, F. Forghieri, A. H. Gnauck, and R. M. Derosier, IEEE J. Lightwave Technol. **13,** 841 (1995).
3. V. E. Zakharov and S. Wabnitz, eds., *Optical Solitons: Theoretical Challenges and Industrial Perspectives* (Springer-Verlag, Berlin, 1999).
4. M. J. Ablowitz and G. Biondini, Opt. Lett. **23,** 1668 (1998).
5. I. R. Gabitov and S. K. Turitsyn, Opt. Lett. **21,** 327 (1996).
6. M. J. Ablowitz, G. Biondini, and E. S. Olson, in *Massive WDM and TDM Soliton Transmission Systems*, A. Hasegawa, ed. (Kluwer Academic, Dordrecht, The Netherlands, 2000), pp. 75–114.
7. J. H. B. Nijhof, N. J. Doran, W. Forysiak, and A. Berntson, Electron. Lett. **34,** 481 (1998).
8. V. E. Zakharov, in *Optical Solitons: Theoretical Challenges and Industrial Perspectives*, V. E. Zakharov and S. Wabritz, eds. (Springer-Verlag, Berlin, 1999), pp. 73–89.
9. V. E. Zakharov and S. V. Manakov, JETP Lett. **70,** 578 (1999).
10. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).
11. N. S. Bergano, C. R. Davidson, M. Ma, A. Pillipetskii, S. G. Evangelides, H. D. Kidorf, J. M. Darcie, E. Golovchenko, K. Rottwitt, P. C. Corbett, R. Menges, M. A. Mills, B. Pedersen, D. Peckham, A. A. Abramov, and A. M. Vengsarkar, in *Optical Fiber Communication Conference*, Vol. 2 of 1998 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1998), postdeadline paper PD12.
12. P. Harper, S. B. Alleston, D. S. Govan, W. Forysiak, I. Bennion, and N. J. Doran, in *Massive WDM and TDM Soliton Transmission Systems*, A. Hasegawa, ed. (Kluwer Academic, Dordrecht, The Netherlands, 2000).
13. R.-M. Mu, T. Yu, V. S. Grigoryan, and C. R. Menyuk, in *Optical Fiber Communication Conference*, Vol. 2 of 1998 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 2000), paper FC1.
14. K. Fukuchi, M. Kakui, A. Sasaki, T. Ito, Y. Inada, T. Tsuzaki, T. Shitomi, K. Fujii, S. Shikii, H. Sugahara, and A. Hasegawa, in *25th European Conference on Optical Communication* (Institute of Electrical and Electronics Engineers, Piscataway, N.J., 1999), paper PD2-10.
15. P. V. Mamyshev and N. A. Mamysheva, Opt. Lett. **24,** 1454 (1999).
16. M. J. Ablowitz and T. Hirooka, Opt. Lett. **25,** 1750 (2000).