Nonlinear chirp of dispersion-managed return-to-zero pulses

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Applying variational methods, we derive a reduced system of equations from the nonlocal equation that governs the average dynamics in dispersion-managed systems. These equations, which apply for any type of return-to-zero pulse, describe the stroboscopic evolution of the pulse parameters and bypass the fast variations inside each dispersion map. In the limit of large map strength we integrate the equations to obtain explicitly formulas for the parameters of a chirped return-to-zero pulse as well as the amount of post-transmission compensation needed to restore the initial pulse width.

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Dispersion management has become a standard technique in long-haul optical fiber communications because of its many beneficial effects on transmission properties. However, the repeated compression–expansion cycles substantially alter the behavior of the pulses and contribute to making the modeling of these systems much more complicated than otherwise would be expected. In a sense, the same features that make dispersion management desirable from a practical point of view contribute to making it difficult to study analytically. A considerable amount of research has been devoted to dispersion management in optical communication systems, and various theoretical approaches have been proposed (see, e.g., Refs. 1–19). Most of these studies have treated dispersion-managed (DM) solitons. In the past few years, however, the attention of system designers has shifted toward more-general return-to-zero (RZ) pulses. The evolution of these RZ pulses is in general nonperiodic (unlike that of DM solitons, which usually require much larger powers). Thus, analytical tools with which to describe transmission in this kind of system have become especially useful.

In this Letter we derive a reduced system of equations that describes the average dynamics of the pulse parameters in systems with strong dispersion management. The equations avoid the complicated dynamics of the pulses inside the dispersion map while at the same time retaining the essential features of the pulse behavior, thus allowing one to focus on the evolution over long distances. In the limit of large map strengths, these equations can be solved exactly, yielding explicit expressions for the evolution of the pulse parameters.

Our starting point is the nonlinear Schrödinger (NLS) equation with loss, amplification, and dispersion management:

$$iu_z + (1/2)d(z)u_{tt} + g(z)\,|u|^2u = 0\,. \tag{1}$$

The dimensionless variables u, z, and t are related to the corresponding quantities in the laboratory frame by $z = z_{\rm lab}/z_*$, $t = t_{\rm ret}/t_*$, $d(z) = -k''(z)/k_*''$, and $u = E/\sqrt{g(z)P_*}$, where E is the electric field envelope, $k_*'' = t_*^{\,2}/z_*$, and z_*, t_*, and P_* are normalization constants. Here we take $t_* = 5$ ps, $P_* = 0.86$ mW, and $z_* = z_{\rm NL} = 500$ km. The functions $d(z)$ and $g(z)$ represent the fiber dispersion coefficient and the power variations that are due to loss and amplification, respectively, both of which we take to be periodic with dimensionless period z_a. These normalizations yield the dispersion coefficient in physical units as $D(z) = d(z) \times 0.04$ ps/(nm/km), and, if amplifiers are placed 50 km apart, $z_a = 0.1$.

To study strong dispersion management we exploit the smallness of z_a and decompose the dispersion coefficient as[2] $d(z) = \langle d\rangle + \Delta(z)/z_a$, where $\Delta(z)$ represents the rescaled zero-mean dispersion variations. (Hereafter, angle brackets denote the average over one period of the dispersion map.) Then a multiple scale expansion of Eq. (1) shows that,[2] to leading order, $\hat u(\omega, z) = \hat U(\omega, z)\exp[-iC(z)\omega^2/2]$, with $C(z) = (1/z_a)\int_0^z {\rm d}z'\Delta(z')$ and where the core pulse shape $\hat U(\omega, z)$ satisfies the DMNLS equation (see also Ref. 3):

$$i\hat U_z - (1/2)\langle d\rangle\omega^2\hat U + \iint {\rm d}\omega'{\rm d}\omega''\hat U(\omega + \omega', z) \times \hat U(\omega + \omega'', z)\hat U^*(\omega + \omega' + \omega'', z)r(\omega'\omega'') = 0\,, \tag{2}$$

where $r(x) = \langle g(z)\exp[iC(z)x]\rangle$ represents the effective nonlinearity averaged over a period of the dispersion map. (In what follows, all integrals are complete unless otherwise noted.) The main advantage of the DMNLS equation compared with the original NLS equation is that it skips the complicated details of pulse evolution inside each dispersion map while retaining most of the characteristic features of DM systems.[4,5]

The DMNLS equation can be derived from the Lagrangian $L = (i/2)\int{\rm d}\omega(\hat U_z\hat U^* - \hat U_z{}^*\hat U) - H$, where the Hamiltonian H is[6]

$$H = \langle d\rangle\int {\rm d}\omega\,\omega^2|\hat U(\omega, z)|^2 - \iiint {\rm d}\omega{\rm d}\omega'{\rm d}\omega''\hat U^*(\omega, z) \times\hat U(\omega + \omega', z)\hat U(\omega + \omega'', z)\hat U^*(\omega + \omega' + \omega'', z)r(\omega'\omega'')\,. \tag{3}$$

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If we introduce a Gaussian ansatz into the above Lagrangian,

$$\hat{U}(\omega, z) = A(z)\exp\{-[\kappa^2(z) - 2ib(z)]\omega^2/4 + i\phi(z)/2\}, \tag{4}$$

the Euler–Lagrange equations of motion yield

$$\kappa' = e_0\kappa^4 I_1\,, \tag{5a}$$

$$b' = \langle d\rangle + e_0\kappa^3(I_0 - \kappa^2 I_2)\,, \tag{5b}$$

$$\phi' = e_0\kappa(5I_0 - \kappa^2 I_2)\,, \tag{5c}$$

where primes denote differentiation with respect to z and

$$I_0(a, b) = \left\langle \frac{g(z)}{\{a^2 + [C(z) + b]^2\}^{1/2}} \right\rangle, \tag{6a}$$

$$I_1(a, b) = -\frac{\partial I_0(a, b)}{\partial b}\,, \qquad I_2(a, b) = -\frac{\partial I_0(a, b)}{\partial a}\,, \tag{6b}$$

with $a = \kappa^2/2$. The pulse energy is $E = A^2/(\sqrt{2\pi}\,\kappa) =$ constant $= 4\sqrt{\pi}\,e_0$, the peak amplitude in the time domain is $A/[\sqrt{2\pi}\,(a^2 + b^2)^{1/4}]$, the rms pulse width is $\tau_{\mathrm{rms}} = (\kappa/a)\sqrt{a^2 + b^2}$, the rms spectral width is $2/\kappa$, and the rms chirp is $b/[2(a^2 + b^2)]$. Note that $A(z)$ can be obtained from E once $\kappa(z)$ is known. In the original NLS problem all these quantities represent the pulse parameters evaluated at the points where $C(z) = 0$.

Equations (5) constitute a reduced dynamic system that describes the slow (i.e., stroboscopic) evolution of the pulse parameters over many dispersion map periods. They are similar in spirit to the fast equations, obtained through either the variational method[7–15] or the rms moment method,[16–18] that describe the full evolution of the pulse parameters. Compared with the fast equations considered in previous papers, however, the slow system of Eqs. (5) has the same advantage that the DMNLS equation has compared with the NLS equation; that is, it bypasses the rapid and complicated internal dynamics of the pulses inside the dispersion map and thus is particularly suited to the study of pulse propagation over long distances. [It is interesting to note that slow equations (5) could also be obtained by averaging of the fast equations over a dispersion map[15].] Equations (5) apply independently of the transmission format. Here we use them to study the behavior of general (i.e., nonsoliton) RZ pulses.

Both the DMNLS equation and Eqs. (5) hold for any choice of dispersion profile $\Delta(z)$. The most practical case is of course that of a two-step dispersion map, where $\Delta(z)$ is the periodic extension of $\Delta(z) = 2s/\theta$ for $|z/z_a| \le \theta/2$ and $\Delta(z) = -2s/(1 - \theta)$ for $\theta/2 \le |z/z_a| \le 1/2$. Here θ is the fraction of the map period that corresponds to the first fiber segment and s is the reduced map strength $s = [(k_1'' - \langle k''\rangle)L_1 - (k_2'' - \langle k''\rangle)L_2]/(4t_*^2) = (\tau_{\mathrm{FWHM}}/2t_*)^2 S$, where S is the conventional map strength.[5,13] Hereafter we assume that the pulse peak power is constant along the line; i.e., we take $g(z) = 1$. In practice, this is a good approximation when there is more than one erbium-doped fiber amplifier (EDFA) per dispersion map or when Raman or distributed amplification is used. In this case the integral I_0 becomes, independently of θ,

$$I_0(a, b) = \frac{1}{2s}\ln\left\{\frac{b + s + [a^2 + (b + s)^2]^{1/2}}{b - s + [a^2 + (b - s)^2]^{1/2}}\right\}. \tag{7}$$

Expressions for $I_{1,2}$ follow from Eq. (6b).

It is not now known whether Eqs. (5) with $I_{0,1,2}$ given by Eqs. (6b) and (7) can be solved analytically. In many practical cases, however, the variations of the dispersion coefficient are quite large. In the limit of strong dispersion maps ($s \gg 1$), the integrals $I_{0,1,2}$ assume a particularly simple form:

$$I_0 \sim (1/s)[\ln(2s/a) + (a^2 - 2b^2)/4s^2] + O(1/s^5)\,, \tag{8a}$$

$$I_1 \sim b/s^3 + O(1/s^5)\,, \qquad I_2 \sim 1/as + O(1/s^3)\,. \tag{8b}$$

Substituting Eqs. (8) into Eqs. (5) with the initial conditions $b(0) = \phi(0) = 0$ and $\kappa(0) = \tau_{\mathrm{rms}}(0) = \tau_0$, we obtain, to leading order,

$$\kappa(z) = \tau_0\,, \tag{9a}$$

$$b(z) = (\langle d\rangle + c_2)z\,, \tag{9b}$$

$$\phi(z) = c_0 z\,, \tag{9c}$$

where

$$c_0 = (e_0\tau_0/s)[5\ln(4s/\tau_0^2) - 2]\,, \tag{10a}$$

$$c_2 = (e_0\tau_0^3/s)[\ln(4s/\tau_0^2) - 2]\,. \tag{10b}$$

It is remarkable that, for large map strengths, the long-term dynamics of a RZ pulse can be described explicitly and in a simple way through Eqs. (9) and (10). From Eq. (9a) we see that, for large s, the spectral width remains unchanged on propagation (consistent with Ref. 4). As the rms pulse width is $\tau_{\mathrm{rms}} = \kappa(1 + 4b^2/\kappa^4)^{1/2}$ the pulse experiences broadening as a result of the nonlinear chirp $c_2 z$, which depends explicitly on pulse energy (through e_0), on initial pulse width τ_0, and of course on map strength s. Note, however, that the dependence of c_2 and of nonlinear propagation constant c_0 on $\ln(s)/s$ (consistent with Refs. 4 and 19) implies a reduction of the effective nonlinearity.

Figure 1 shows a comparison of the analytical results and the numerical simulation of NLS equation (1) for transmission of a 5-ps pulse ($\tau_0 = 1$, for a bit rate of 40 Gbits/s), with $\langle d\rangle = 0.1$ and $s = 20$[$\langle D\rangle = 0.04$ ps/(nm/km) and $\Delta D = 62.5$ ps/(nm/km) for equal-length fiber sections] up to $z = 10$ (that is, over a total distance of 5000 km). The agreement between analytical formulas and numerical simulations is remarkable. Some differences exist for the chirp and the spectral width if the loss is compensated for by one EDFA per map. However, the deviations become much smaller for stronger maps or if more than one EDFA or Raman amplifier is used. Obviously Eqs. (5)–(8) still hold if a prechirp $b_0 \neq 0$ is applied to the initial pulse and the only change

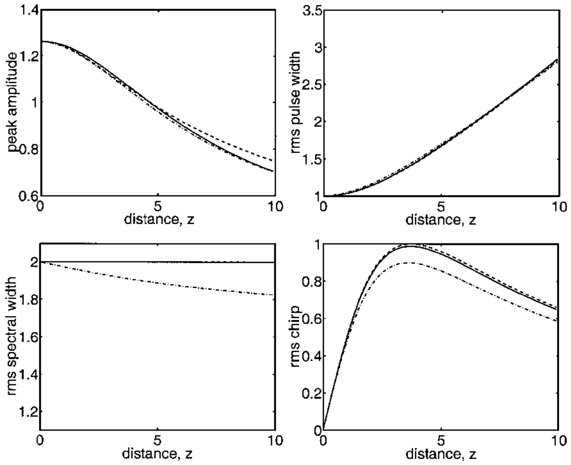

Fig. 1. Evolution of the pulse parameters for a Gaussian input with $\langle d \rangle = 0.1$, $s = 20$, $\tau_0 = 1$, and $E = 2$. Dashed curves, Eqs. (9); solid curves, numerical simulations of the NLS equation with $g(z) = 1$; dotted–dashed curves, NLS equation with one EDFA per map. [The spectral width for $g(z) = 1$ is indistinguishable from the analytical curve.]

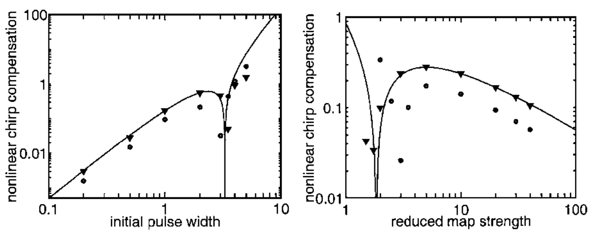

Fig. 2. Chirp compensation needed at $z = 10$ as left, a function of τ_0 with $s = 20$ and right, a function of s with $\tau_0 = 1$ for $\langle d \rangle = 0$ and $E = 2$. Solid curves, Eqs. (10); triangles, numerical results, $g(z) = 1$; circles, numerical results, one EDFA per map.

is the additional constant b_0 that appears on the right-hand side of Eq. (9b). Note also from Eq. (9b) that the effects of a nonzero average dispersion $\langle d \rangle$ are decoupled from the nonlinearity and can easily be accounted for by post-transmission compensation.

Figure 2 shows the amount of post-transmission compensation that should be applied to the pulse after propagation to $z = 10$ to restore its rms width to the initial value τ_0 (or as close as possible to it) as a function of both s and τ_0 itself. [If desired, a larger compression at the receiver could be engineered on the basis of Eqs. (10) and prechirp b_0]. The cusps correspond to a change of sign of c_2, which is paralleled by numerical data. Again, the agreement with the lossless case is excellent, except at low values of s or large values of τ_0. In fact, the formulas for large map strength compare favorably even when the value of s is as low as 2. The qualitative behavior with one EDFA per map also mimics the analytical curves, although the actual values differ somewhat, with the largest differences seemingly arising from a displacement of the zero of c_2. When $g(z) = 1$, the zero of c_2 also seems to delimit the region in which the large s limit provides a good approximation to the system. The discrepancies for large τ_0 are due to the fact that, for fixed s, effective map strength S is inversely proportional to the square of the pulse duration τ_0.

In conclusion, we have derived a set of equations to describe the evolution of the characteristic parameters of a RZ pulse in the presence of dispersion management, loss, and amplification. The equations highlight the average dynamics over long distances and circumvent the complexities of pulse evolution within each dispersion map. In the limit of large map strengths we have integrated the equations exactly, obtaining analytic expressions for the evolution of the pulse parameters. These expressions show the explicit dependence of the nonlinear chirp and the nonlinear phase on the pulse energy, the map strength, and the initial pulse width and should prove to be a useful modeling tool for the study of quasi-linear dispersion-managed RZ transmission systems.

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