FOCUS ISSUE: Optical Solitons—Perspectives and Applications

Optical solitons: Perspectives and applications

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This article serves as an introduction to the focus issue on optical solitons. After a short review of the history of solitons and the field of integrable systems, a brief overview of the development of nonlinear optics and optical solitons is provided. Next, the various contributions to this focus issue are presented, and a few separate remarks are devoted to optical communications, where solitons promise to play a decisive role in the next generation of commercial systems. © *2000 American Institute of Physics.* [S1054-1500(00)02303-X]

Interest in optical solitons has grown steadily in recent years. The field has considerable potential for technological applications, and it presents many exciting research problems both from a fundamental and an applied point of view. New optical devices are in various stages of development; soliton information processing looms on the horizon. At the same time, basic research in nonlinear optical phenomena maintains its vitality. In the last decade, many distinguished physicists and applied mathematicians have contributed to some or all of these areas. However, to a large extent the various communities involved—engineers, physicists and mathematicians—still operate within their own boundaries. One of the purposes of this volume is to feature articles which bring together different perspectives, with the hope of stimulating future interaction between researchers with different backgrounds. In this introductory article we take the occasion to briefly review the parallel development of solitons and nonlinear optics before introducing the various articles contained in this volume.

SOLITONS AND NONLINEAR PHENOMENA

The study of physical phenomena by means of mathematical models is an essential element in both theoretical and applied sciences. Often such models lead to nonlinear systems, and, in a surprisingly large number of cases, to certain prototypical equations. Solitons are an intrinsically nonlinear phenomenon, and their history is intimately connected to the development of the theory of nonlinear wave equations.

The first observation of a solitary wave occurred in August 1834, when Scottish engineer Scott Russell described a large ''wave of translation,'' propagating for miles in a narrow barge canal.[1] Russell's discovery generated a lively controversy as to whether the inviscid equations of water waves would possess such solutions. Several people contributed to the effort of trying to understand the phenomenon, including Airy,[2] Boussinesq[3] and Korteweg and de Vries (KdV), who in 1985 derived and studied a nonlinear wave equation—which now bears their name—as a model for the evolution of long waves in a shallow one-dimensional water channel.[4] The controversy was then resolved when it was shown that the KdV equation admits solitary wave solutions.

It would be almost seventy years before these findings would be taken any further. In the meantime, other problems appeared which would eventually be connected to Russell's discovery. In 1847 Stokes had obtained a series solution to the water wave equations, describing a periodic nonlinear wave train in deep water.[5] The series was formally proven to be convergent by Levi Civita in 1926.[6] However, Benjamin and Feir showed in 1967 that the Stokes wave (and in fact any uniform wave train in sufficiently deep water) is modulationally unstable.[7] Meanwhile, in 1965 Fermi, Pasta and Ulam numerically integrated the ordinary differential equations which describe a set of coupled anharmonic oscillators—now called the Fermi–Pasta–Ulam (FPU) problem—in order to understand the mechanisms which lead to thermal equilibrium. The equipartition of energy suggested that the nonlinearity would quickly cause the energy

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contained in the initial condition to be redistributed among all the modes, thus driving the system to equilibrium. However, to their great surprise, they found that only a very small number of modes were actually participating in the dynamics, and that the energy flowed back and forth among them in a recurrent pattern.[8]

The explanation of all these effects came with a breakthrough in our understanding of nonlinear phenomena. In 1965 Zabusky and Kruskal, while studying the FPU problem, re-derived the KdV equation as a continuum approximation, and they discovered through numerical simulations that the solitary wave solutions to the KdV equation had a property which was previously unknown: namely, the fact that these solutions interacted ''elastically'' with each other.[9] Because of this particle-like property, they termed these solutions *solitons*. Shortly after this discovery, Gardner, Greene, Kruskal and Miura pioneered what was to develop into a new method of mathematical physics by solving the initial value problem for the KdV equation using the ideas of direct and inverse scattering.[10] One year later, Lax generalized these ideas,[11] and in 1971 Zakharov and Shabat showed that the method worked for another physically significant nonlinear evolution equation, the nonlinear Schrödinger (NLS) equation[12]—which is the underlying mechanism for the Benjamin–Feir instability in water waves. Then in 1974 Ablowitz, Kaup, Newell and Segur showed how those techniques could be used to solve a wide class of nonlinear evolution equations. They also showed that these methods are the analog of the Fourier transform for nonlinear problems.[13] They called the procedure the *inverse scattering transform* (IST).

Since then the theory of solitons and integrable systems has blossomed into a rich and diverse field, and it would be impossible to do it justice in such a short space as this. We refer the interested reader to well-known monographs[14–19] for further details on the subject.

In passing, we mention that the term ''soliton'' is used in optics to describe a localized pulse that travels without a change in shape, an entity which the mathematics community refers to as a solitary wave. Historically speaking, solitons were defined as those solutions which asymptotically preserve their speed and amplitude upon interaction. They correspond to the poles of the scattering coefficient in the underlying scattering problem, and as such only exist for integrable problems (that is, problems solvable by IST), while solitary waves exist for a much broader class of nonlinear wave equations. Nonetheless, it is quite common, especially in the optics community, to refer to these solitary wave pulses as solitons, and we will conform to this convention.

NONLINEAR OPTICS AND OPTICAL SOLITONS

The realm of nonlinear optics consists of those phenomena for which the optical properties of a material depend upon the strength of the applied field. Typically, only laser light is sufficiently intense to modify the optical properties of a system. Therefore it is not a coincidence that the beginning of nonlinear optics is traditionally taken to be 1961, when, just one year after the invention of the laser,[20] Franken, Hill, Peters and Weinreich reported the generation of a second harmonic by focusing a laser beam through a crystalline quartz.[21] After this demonstration, several experiments followed in rapid succession. The generation of the sum of two different frequencies was achieved by the same group in 1962,[22] difference frequency by Smith and Braslau in the same year,[23] optical rectification was reported by Ward and Franken in 1964[24] and the first optical parametric oscillator was demonstrated by Giordamaine and Miller in 1965.[25] Meanwhile, Armstrong, Bloembergen, Ducuing and Pershan published a comprehensive study[26] including the calculation of nonlinear susceptibilities and a complete theory for the interactions of two and three electromagnetic waves. Pioneering work on these problems was also collected in a book by Akhmanov and Khokhlov.[27]

The previous effects manifest themselves in quadratic media, that is, media where the material response depends on the square of the amplitude of the electric field. During the same years, a number of authors also considered propagation of an intense laser beam in a cubic medium, including Chiao, Garmire and Townes,[28] Kelley,[29] Talanov[30] and Ostrovsky.[31] In this case the nonlinearity results in an intensity-dependent contribution to the index of refraction, which is responsible for the self-focusing of the optical beam. The evolution of the beam profile in such media was then found to be governed by a multi-dimensional NLS equation. From a more general point of view, Benney and Newell showed that the NLS appears as a universal equation which describes the slow modulation of a weakly nonlinear wave packet.[32]

Several other physical effects were discovered within a short period. As a result, nonlinear processes acquired increasing importance in optics, and eventually evolved into an independent field, whose interest has grown rapidly. Its scope now ranges from fundamental studies of the interaction of light with matter to technological applications such as frequency conversion and optical switching. Depending on the particular physical situation, the fundamental physical phenomena possible include multiphoton absorption and emission, generation of harmonics, parametric effects, stimulated scattering, generation of highly coherent and highly nonstationary radiation, the occurrence of field-dependent optical material parameters, optical bistability, phase conjugation and saturation phenomena.

One of the areas of nonlinear optics which has attracted considerable attention in the last twenty years is the field of soliton communication systems: we devote the next section to this specific topic. Many technological applications of optical solitons are being actively pursued, including soliton switching, pulse compression and wavelength conversion. Corresponding physical problems which feature optical solitons are second-harmonic generation, three-wave interactions, self-induced transparency, together with gap solitons, incoherent solitons, etc. Some of these areas are discussed in the articles contained in this volume; for the others we refer the reader to the many books[33–38] and review articles[39–42] on the subject.

This focus issue includes both review articles and original results. The contributions can roughly be classified in two broad categories: a first group of papers is devoted to fiber-optic solitons, which we discuss separately in the next section. The second group of papers spans several areas. The first three articles in this group are devoted to propagation phenomena which go beyond the limits of the NLS equation. The article by Gromov and Talanov deals with sub-picosecond solitons in fibers; Moloney *et al.* present a review of the physics of femtosecond pulses propagating in air, and Blair offers a detailed study of nonparaxial effects for spatial solitons. Two articles are devoted to gap solitons: Aceves provides an extensive review of the topic, while Trillo *et al.* focus on gap solitons in quadratic media. The remaining three articles in this volume deal with somewhat different problems: Akhmediev and Ankiewicz present a review of multi-soliton complexes; Cundiff, Collings and Bergman report on the experimental observation of a new type of solitons, so-called polarization-locked solitons; finally, Paniou *et al.* study the effect of symmetric perturbations of a two-soliton solution of NLS.

OPTICAL COMMUNICATIONS

Over the past thirty-five years, soliton research has been conducted in fields as diverse as particle physics, molecular biology, quantum mechanics, geology, meteorology, oceanography, astrophysics and cosmology. But the area of soliton research which is technologically most significant is currently the study of solitons in optical fibers, where the sought-after goal is to use soliton pulses as the information carrying ''bits'' in optical fibers. Optical communications is a wonderful example of how the interplay between mathematics and more applied research has generated significant technological advances: Historically, soliton research and optical communications started as two separate fields. In the last twenty years, however, technological advances in conventional transmission systems have often inspired parallel developments in soliton systems and *vice versa*, up to the point where the two transmission techniques have become so similar that they can be considered just as two different realizations of the same type of system—the nonlinear optical transmission line.[43]

Hasegawa's article in this issue provides an excellent introduction to the history of soliton communication systems, including its history, and we will not repeat it here. Several monographs and reviews are also available.[37,38,44–46] The rate at which data can be transmitted has increased by four orders of magnitude over the last decade alone; Nakazawa's article provides an account of soliton transmission experiments performed in recent years at NTT laboratories.

One of the most striking developments for soliton transmission systems in the last few years was the introduction of dispersion management. This is the practice of alternating fibers with opposite sign of chromatic dispersion along the transmission line, a technique already well-known for conventional systems.[47] Mathematically, this translates into large and rapidly varying terms in the NLS equation, which alter dramatically its structure and its solutions. These solutions, so-called ''dispersion-managed'' solitons, are characterized by a number of remarkable properties which have no analog for ''classical'' NLS solitons. From a fundamental point of view, these properties arise from the fact that—unlike the case when small perturbations are considered—the asymptotic behavior of a system with strong dispersion management is not described by the NLS equation; instead, the dynamics is asymptotically governed by a nonlocal equation which is found by averaging the NLS over the large and rapid variations of the fiber dispersion.[48,49] A number of techniques have been developed in the last few years to study systems with strong dispersion management; the article by Cautaerts, Maruta and Kodama contains an overview of these techniques and a discussion of their relative merits.

The recent development of optical fiber fabrication techniques that suppress the water absorption peak in the 1400 nm wavelength region[50] opens up the possibility of massive wavelength-division multiplexing across the whole wavelength range from 1200 to 1600 nm. Two complementary ways to exploit this enormous capacity consist of increasing the number of channels and increasing the single-channel bit-rate. However, in both cases the difficulties facing the designers of communication systems are basically the same, and arise from the interplay of four basic impairments:[51] (i) chromatic dispersion; (ii) nonlinearity; (iii) amplifier noise; and (iv) birefringence effects, which are also called polarization-mode dispersion (PMD). The first three effects have been extensively studied, at least for classical solitons. The last one has gained considerable attention in the last few years, and is rapidly becoming one of the main factors which limit the performance of transmission systems at very high bit rates. The remaining two articles devoted to soliton communications deal with polarization effects: Lakoba and Pelinovky study the existence of internal modes in scalar and vector DM solitons, while Chen and Haus discuss the effect of PMD on the soliton solutions of the integrable coupled NLS equation (i.e., the Manakov system[52]).

CONCLUDING REMARKS

Optical solitons have established themselves as a central presence in the study of physical phenomena. The field is still evolving at a very rapid pace, and we will not try to predict the next developments. Whatever directions the field will take, however, we are confident that it will continue to offer exciting topics from both a mathematical and from a physical point of view, as well as providing problems which have direct relevance for concrete technological applications. We hope that this volume will provide a reference tool as well as an introduction for scientists interested in this topic.

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