MTH 628 ALGEBRAIC TOPOLOGY

Homework 5 (due Thu. 2017.05.11)

1. The following fact is true (and you don't need to prove it): if G is an abelian group such that $\mathrm{Hom}(G, \mathbb{Z}) = 0$ and $\mathrm{Ext}(G, \mathbb{Z}) = 0$ then $G = 0$. Let $f\colon X \to Y$ be a map of spaces. Show that the following conditions are equivalent:

(i) $f_*\colon H_n(X) \to H_n(Y)$ is an isomorphism for all $n \geq 0$.
(ii) $f^*\colon H^n(Y, \mathbb{Z}) \to H^n(X, \mathbb{Z})$ is an isomorphism for all $n \geq 0$.

2. Let X be a finite CW complex. Show that $H^n(X, \mathbb{Q}) \cong H_n(X, \mathbb{Q})$ for all n.

3. Let X be a path connected space, and let $x_0 \in X$. Show that for any abelian group G we have an isomorphism $H^1(X, G) \cong \mathrm{Hom}(\pi_1(X, x_0), G)$.

4. Let X be a space and let G be an abelian group. Notice that we have function:

$$\Phi\colon C^n(X; G) \times C_n(X) \to G$$

given by $\Phi(\varphi, x) = \varphi(x)$. Show that this induces a well-defined function

$$\Phi^*\colon H^n(X, G) \times H_n(X) \to G$$