MTH 628 ALGEBRAIC TOPOLOGY

Homework 3 (due Thu. 2017.04.06)

1. Let $f, g\colon S^n \to S^n$ be maps such that $f(x) \neq g(x)$ for all $x \in S^n$. Show that $\deg(f) = (-1)^{n+1}\deg(g)$.

2. Let $f\colon S^n \to S^n$ be a map homotopic to a constant map. Show that there exist points $x, y \in S^n$ such that $f(x) = x$ and $f(y) = -y$.

3. Let X be a space, and let $U_1, \dots, U_n \subseteq X$ be open sets such that $X = \bigcup_{i=1}^{n} U_i$. Assume that each intersection $U_{i_1} \cap \dots \cap U_{i_k}$ is either empty or contractible. Show that $\widetilde{H}_q(X) = 0$ for all $q \geq n-1$.

4. Let X be an n-dimensional CW-complex. Show that there exists a point $x \in X$ such that $H_n(X, X - \{x\}) \neq 0$.