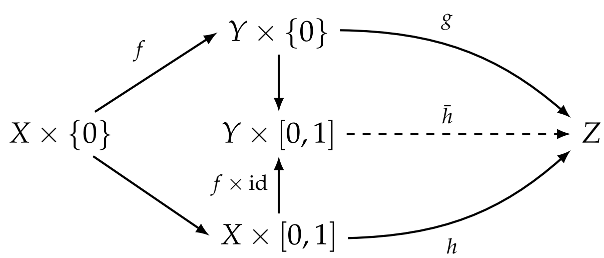
MTH 628 ALGEBRAIC TOPOLOGY

Homework 2 (due Thu. 2017.03.09)

1. Let $A \subseteq X$. Recall that the inclusion $i\colon A \hookrightarrow X$ is a cofibration if any map $h\colon X \times \{0\} \cup A \times [0,1] \to Z$ extends to a map $\bar{h}\colon X \times [0,1] \to Z$. Show that $i\colon A \hookrightarrow X$ is a cofibration if and only if $X \times \{0\} \cup A \times [0,1]$ is a retract of $X \times [0,1]$.

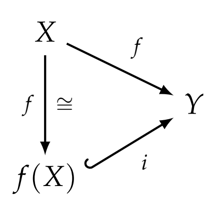
2. Show that if X is a Hausdorff space, and $i\colon A \hookrightarrow B$ and $j\colon B \hookrightarrow X$ are cofibrations then $ji\colon A \hookrightarrow X$ is a cofibration.

3. Here is a more general definition of a cofibration. Let $f\colon X \to Y$ be a map of spaces. We will say that f is a cofibration if for any maps $g\colon Y \to Z$ and $h\colon X \times [0,1] \to Z$ satisfying $h(x,0) = gf(x)$ there is $\bar{h}\colon Y \times [0,1] \to Z$ such that the following diagram commutes:



Notice that if f is an inclusion of a subspace into a space then we recover the original definition of a cofibration.

Show that if $f\colon X \to Y$ is a cofibration in the sense of the above general definition then $f\colon X \to f(X)$ is a homeomorphism. This means that any cofibration is, up to a homeomorphism, an inclusion of subspace :



4. Show that for any space X there is an isomorphism:

$$H_q(X \times S^1) \cong H_q(X) \oplus H_{q-1}(X)$$