MTH 628 ALGEBRAIC TOPOLOGY

Homework 1 (due Thu. 2017.02.23)

1. Recall that if $\omega\colon [0,1] \to X$ is a path in X then $\bar{\omega}$ is the path given by $\bar{\omega}(t) = \omega(1-t)$. Recall that as in the Hurewicz theorem we can consider each path in X as a singular 1-simplex in X by using a homeomorphism $\Delta^1 \cong [0,1]$. Show that in $C_1(X)$ we have:

$$\bar{\omega} = -\omega + b$$

for some $b \in B_1(X)$.

2. Let C_*, D_* be chain complexes and $f_*, g_*, h_*\colon C_* \to D_*$ be chain maps. Show that if $f_* \simeq g_*$ and $g_* \simeq h_*$ then $f_* \simeq h_*$ (where $f_* \simeq g_*$ means that there is a chain homotopy between f_* and g_*).

3. Let C_*, D_*, E_* be chain complexes and $f_*, f'_*\colon C_* \to D_*$, $g_*, g'_*\colon D_* \to E_*$ be chain maps. Show that if $f_* \simeq f'_*$ and $g_* \simeq g'_*$ then $g_* f_* \simeq g'_* f'_*$.

4. Recall that a subspace $A \subseteq X$ is a retract of X if there exists a map $r\colon X \to A$ such that $r(x) = x$ for all $x \in A$. Show that if $A \subseteq X$ is a retract of A and the inclusion $i\colon A \hookrightarrow X$ is a cofibration then $\widetilde{H}_n(X) \cong \widetilde{H}_n(A) \oplus \widetilde{H}_n(X/A)$ for all $n \geq 0$.