(b)(i) $A=1,\ B=-1,\ C=6,\ D=0 \Rightarrow \frac{d^2y}{dt^2}-\frac{dy}{dt}=6y$

$$\Rightarrow \frac{d^2y}{dt^2}-\frac{dy}{dt}-6y=0$$

A homogeneous linear 2nd-order ODE with constant coefficients. Solve by substituting $y(t)=e^{rt}$.

The characteristic equation is: $r^2-r-6=0$

$$r_{1,2}=\frac{1\pm\sqrt{1+24}}{2}=\frac{1\pm5}{2} \qquad r_1=3 \qquad r_2=-2$$

Two distinct real roots.

$\Rightarrow$ The general solution is: $y(t)=C_1e^{3t}+C_2e^{-2t}$

(ii) Use IC to find C_1, C_2.

$y(0)=2=C_1+C_2$

$y'(t)=3C_1e^{3t}-2C_2e^{-2t} \Rightarrow y'(0)=3=3C_1-2C_2$

$3C_1-2(2-C_1)=3 \Rightarrow 5C_1=7 \Rightarrow C_1=\frac{7}{5}$

$C_2=2-C_1=\frac{3}{5}$

The solution to the IVP is:

$$y(t)=\frac{7}{5}e^{3t}+\frac{3}{5}e^{-2t}$$