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The Wooden Tablets from Cairo: The Use of the Grain Unit *ḥḳ3t* in Ancient Egypt

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First of all, I would like to express my thanks to Professor Dr. Fayza Haikal who sent me the photos of the Cairo wooden tablets and thus enabled me to deal with the notable calculations which are recorded on them.

Introduction

The *ḥḳ3t* unit was used by ancient Egyptians for determination of the amount of grain. It occurs in texts of economic character, in lists of products or payments (either in tombs or in papyri). Data in *ḥḳ3t* units can also be found in mathematical texts, especially in problems concerning granaries[1], the so-called bread and beer problems[2] and several other types of mathematical examples[3].

Without any doubt, every ancient Egyptian scribe should have been perfectly familiar with the system of the *ḥḳ3t* unit. He should have been able to express various fractions of a *ḥḳ3t* by the so-called "fractions of the eye of Horus" and to accomplish transfers between the two systems, either automatically or with the help of some kind of tables. Two wooden tablets from Cairo museum contain very interesting calculations which may help us to explain how the *ḥḳ3t* system was used by the ancient Egyptians.

The wooden tablets

The two tablets which are the focus of this article, were published by G. Daressy in *Catalogue générale*[4]. He gave there basic information about both tablets, mainly their dimensions and condition and briefly described the hieratic notes. A few years later, Daressy wrote an article dealing with these tablets in more detail. He translated the calculations and tried to explain them. He described them as multiplications of integers and fractions.[5]

[1] Problems 41–46 of the Rhind papyrus.

[2] Problems 5, 8, 9, 12, 13, 15, 16, 20, 21, 22, 24 of the Moscow papyrus and problems 69–78 of the Rhind papyrus.

[3] For example problems 47, 64, 66, 68 of the Rhind papyrus.

[4] Daressy, G.: *Catalogue générale des Antiquités égyptiennes du Musée de Cairo, Ostraca*, Cairo 1901, pp. 95-96.

[5] Daressy, G.: "Calculs égyptiennes du Moyen-empire", *RecTrav* 28 (1906), pp. 62-72.

Thomas E. Peet also directed his attention to these tablets. He was the first to realize that there were two sorts of fractions used in the calculations of the tablets, the "ordinary" fractions and the so-called "fractions of the eye of Horus" which form the *ḥkꜣt* system. Thus, it came to light that the calculations which he briefly described were closely connected with the transfers of ordinary fractions to the system of the *ḥkꜣt* unit.[6]

This article aims to give a complete overview of the calculations and tries to demonstrate a possible connection between mathematics and practical experience of the ancient Egyptian scribes.

The wooden tablets which were found at Akhmim and are dated to the Middle Kingdom, are today deposited in the Cairo museum as n. 25367 and n. 25368. Both tablets are plastered and polished, so that they were convenient for writing. The tablet n. 25367 measures 46.5 by 26 cm and contains remnants of a letter which is unfortunately very much destroyed and a list of names of 31 *smdt* people dated to the 28th year of the reign of an unknown king. On the other side of the tablet, there are the calculations which will be described below. The second tablet n. 25368, measuring 47.5 by 25 cm, contains a list of 27 *smdt* people. The rest of the surface of the tablet is completely covered with calculations.[7]

The calculations

Before explaining the calculations of the wooden tablets, let us describe the basic regularities of the *ḥkꜣt* system which will enable us to better follow and understand the calculations themselves.

The *ḥkꜣt*, written as , var. corresponds to 4.805 litres and was used by the ancient Egyptians as a grain unit. Small amounts of grain were expressed by means of a fixed system of a limited number of fractions which were represented by special signs: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$. This system is known as "fractions of the eye of Horus", since the signs when put together, form the sign representing the Horus eye *wḏꜣt*. According to Möller, the signs for the fractions of the Horus eye must be very old[8]. The earliest evidence for their use comes from the fifth dynasty, but the signs must have been invented much earlier.

For extremely small amounts of grain which could not be expressed even by fractions as $\frac{1}{64}$ of a *ḥkꜣt*, a smaller unit called *rꜣ* was developed. A whole *ḥkꜣt* contained 320 *rꜣ*, so that 5 *rꜣ* was equal to $\frac{1}{64}$ of a *ḥkꜣ*t.

A double-*ḥkꜣt*, a quadruple-*ḥkꜣt* and a hundred-*ḥkꜣt* were used to determine large amounts of grain, for example those deposited in granaries. These units do not occur in the Cairo wooden tablets.[9]

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[6] Peet, T. E.: "Arithmetic in the Middle Kingdom", *JEA* 9 (1923), pp. 91-95.

[7] After Daressy, *Catalogue générale.*

[8] Möller, G.: "Die Zeichen für die Bruchteile des Hohlmaßes und das Uzatauge", *ZÄS* 48 (1911), pp. 99-105.

[9] For more information about the *ḥkꜣt* system see: Möller, *op. cit.*; Reineke, W. F.: "Der

Every fraction which was not one of the "fractions of the eye of Horus" mentioned above must have been substituted by a combination of these, for example $\frac{1}{3}$ of a *ḥḳ3t* was expressed as ($\frac{1}{4}$ + $\frac{1}{16}$ + $\frac{1}{64}$) of *ḥḳ3t* + (1 + $\frac{2}{3}$) of *r3*. In the Cairo tablets we can find complete processes of such transfers for $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{13}$ of a *ḥḳ3t*. As we can see below, two different methods of transferring a fraction into a *ḥḳ3t* system can be distinguished, one of them was used only for $\frac{1}{3}$, all the other fractions were transferred by the use of the second method.

A complete list of the calculations follows now. Most of the examples are repeated several times on the wooden tablets. We will mention every occurrence of these calculations since we do not only want to describe the method of reckoning, but also need to show some special features of the individual examples as evident mistakes and scribal errors, which we consider to be extremely interesting.

Determining a $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{13}$ of a *ḥḳ3t* will be introduced one after another. The best preserved example of every calculation will be described in detail at first; in case of the other ones, only the existing differences will be mentioned. The actual position of the individual examples on the wooden tablets is marked in the plates.

In the examples, the numbers in the left column tell which multiples of the subject of reckoning occur in the corresponding lines of the right column. In the right column, two sorts of values appear. The fractions of the *ḥḳ3t* unit are written in italic to express that these were represented by signs of "fractions of the eye of Horus", and values in *r3* are written by "ordinary" signs. We would expect the sign + between the values in the right column to understand exactly what the record means; however, no such signs were used by ancient Egyptians.

The results of the operations undertaken in the calculations are never designated, they always occur as object of further reckoning. The total results of the transfers expressed by means of the *ḥḳ3t* system, can be found in the first lines of tests of correctness (see below).

Calculation for $\frac{1}{3}$ of a *ḥḳ3t* occurs twice on the tablets. Neither calculation contains any mistakes or scribal errors. ← NOTE ROUNDING ERRORS. OK

The procedure of the transfer of $\frac{1}{3}$ of a *ḥḳ3t* to the system of "fractions of the eye of Horus" can be divided into three parts, which can be described as follows: the first part consists of the determination of $\frac{1}{3}$ of 5, the second part is a multiplication of the result which was reached in the first step until $\frac{1}{3}$ of a *ḥḳ3t* is found, the third part of the calculation is a test whether the result achieved in the second step is correct. The explanation of this procedure will be introduced below.

Zusammenhang der altägyptischen Hohl- und Langenmaße", *MIO* 9 (1959), pp. 145-163.

25368	6:	1	$\frac{1}{3}$
		2	$\frac{2}{3}$
		4	$1\ \frac{1}{3}$
		$\frac{1}{64}$	$1\ \frac{2}{3}$
		$\frac{1}{32}$	$3\ \frac{1}{3}$
		$\frac{1}{16}$	$\frac{1}{64}\ 1\ \frac{2}{3}$
		$\frac{1}{8}$	$\frac{1}{32}\ 3\ \frac{1}{3}$
		$\frac{1}{4}$	$\frac{1}{16}\ \frac{1}{64}\ 1\ \frac{2}{3}$
		$\frac{1}{2}$	$\frac{1}{8}\ \frac{1}{32}\ 3\ \frac{1}{3}$
		\1	$\frac{1}{4}\ \frac{1}{16}\ \frac{1}{64}\ 1\ \frac{2}{3}$
		\2	$\frac{1}{2}\ \frac{1}{8}\ \frac{1}{32}\ 3\ \frac{1}{3}$

1. $\frac{1}{3} \times 5 \; <= 1 + \frac{2}{3} >$

The first step, which consists of the first three lines, results in the value of a third of 5.

Adding a third with its quadruple, which is $1 + \frac{1}{3}$, we get the result $1 + \frac{2}{3}$. This value is not marked as the result, it occurs first as the object of doubling at the second part of the calculation.

The number 5 of this multiplication is in *r3* units and since we know that 5 *r3* correspond to $\frac{1}{64}$ of a *ḥḳ3t* we can say that a third of $\frac{1}{64}$ of a *ḥḳ3t* was found, as we can see in the fourth line.

2. From the fourth to the ninth lines, a doubling of the value found in step 1 was put through until a value corresponding to a third of a *ḥḳ3t* was reached.

A double of $1 + \frac{2}{3}$ is $3 + \frac{1}{3}$ which corresponds to a third of $\frac{1}{32}$ of a *ḥḳ3t*. Doubling this, we get $6 + \frac{2}{3}$ of *r3* and since we know that 5 *r3* is $\frac{1}{64}$ of a *ḥḳ3t* we can express it as $\frac{1}{64}$ of a *ḥḳ3t* + $(1 + \frac{2}{3})$ of *r3*. Further doubles are reckoned in the same way.

In the left column of this multiplication, there are all of the fractions which form the *ḥḳ3t* system. Since their sum makes $\frac{63}{64}$ of a *ḥḳ3t* the corresponding values of the right column should be added with the value corresponding to $\frac{1}{64}$ to get a third of a whole *ḥḳ3t*. On the other hand, we do not have to reckon in such a complicated way, namely adding all the values from the right column with one more $1 + \frac{2}{3}$. Since the last multiple is a half as we can see in the left column, we can easily double the value in the right column once more to get the total result. Thus, the result of step 2, that means a third of a *ḥḳ3t* which is

$(\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ of *ḥḳꜣt* + $(1 + \frac{2}{3})$ of *rꜣ*, appears in the line which opens the last part of the calculation, step 3.

3. $3 \times [(\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ *ḥḳꜣt* + $(1 + \frac{2}{3})$ *rꜣ*] < = 1 *ḥḳꜣt* >

The test of the correctness of the result reached in step 2 consists of multiplication of the value $(\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ of *ḥḳꜣt* + $(1 + \frac{2}{3})$ of *rꜣ* by 3 to show that $\frac{1}{3}$ of a *ḥḳꜣt* times 3 is really 1 *ḥḳꜣt*. Adding the values of the fractions in the *ḥḳꜣt* unit, we get $\frac{63}{64}$ of a *ḥḳꜣt*, the sum of the other values is 5 *rꜣ*, that means $\frac{1}{64}$ of a *ḥḳꜣt*. The total is really 1 *ḥḳꜣt*.

25368 12:	1	$\frac{1}{3}$
	2	$\frac{2}{3}$
	4	$1\ \frac{1}{3}$
	$\frac{1}{64}$	$1\ \frac{2}{3}$
	$\frac{1}{32}$	$3\ \frac{1}{3}$
	$\frac{1}{16}$	$\frac{1}{64}\ 1\ \frac{2}{3}$
	$\frac{1}{8}$	$\frac{1}{32}\ 3\ \frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{16}\ \frac{1}{64}\ 1\ \frac{2}{3}$
	$\frac{1}{2}$	$\frac{1}{8}\ \frac{1}{32}\ 3\ \frac{1}{3}$
	\1	$\frac{1}{4}\ \frac{1}{16}\ \frac{1}{64}\ 1\ \frac{2}{3}$
	\2	$\frac{1}{2}\ \frac{1}{8}\ \frac{1}{32}\ 3\ \frac{1}{3}$

The second occurrence of the same calculation follows exactly the same procedure as the one described above.

The second method of determining a certain value of a fraction of *ḥḳꜣt* is easier and can be described as follows. First, a *ḥḳꜣt* in the form of 320 *rꜣ* is multiplied by the given fraction, that means that it is divided by the denominator of the fraction. According to the Egyptian method of division, in fact, the denominator is multiplied until the value of 320 *rꜣ* is reached. In most cases, the division is done through multiplying the value of the denominator of the fraction by 10 and by further doubling. The result of the division is never explicitly mentioned. It seems to be transferred from the *rꜣ* into the *ḥḳꜣt* system automatically and in this form it appears as the subject of the reckoning in step 2.

In the second part of the calculation, the result is multiplied by the value of the denominator of the given fraction to test whether it is correct. The result of this test is also never mentioned, but we can calculate ourselves, that 1 *ḥḳꜣt* is always found.

There are four copies of the calculation for $\frac{1}{7}$ of a *ḥḳꜣt*. Each of them contains scribal errors as we can see below.

25368 13:	1	7
	10	70
	20	140
	40	280
	2	12^{sic}
	4	24^{sic}
	$\frac{1}{7}$	1
	$\frac{1}{4}$ $\frac{1}{28}$	2
	$\frac{1}{2}$ $\frac{1}{14}$	4
	1	$\mathit{\frac{1}{8}}$ $\mathit{\frac{1}{64}}$ $\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{14}$
	2	$\mathit{\frac{1}{4}}$ $\mathit{\frac{1}{32}}$ 1 $\frac{1}{4}$ $\frac{1}{7}$ $\frac{1}{28}$
	[4	$\mathit{\frac{1}{2}}$] $\mathit{\frac{1}{16}}$ 2 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{14}$ [$\frac{1}{28}$]

1. $320 \div 7 <= 45 + \frac{1}{2} + \frac{1}{7} + \frac{1}{14}>$

Within the division of 320 *r3* by 7, the scribe has already made an error in the fifth line. As the double of 7, the value 12 is stated instead of 14. In the next step, this mistake is continued as 24 instead of 28 for the quadruple of 7. As we can see below, this error occurs in every copy of this calculation on the tablets. However, it does not seem to have influenced the result of the calculation, since that itself is correct. The result of the division which is $(\mathit{\frac{1}{8}} + \mathit{\frac{1}{64}})$ of *ḥḳ3t* + $(\frac{1}{2} + \frac{1}{7} + \frac{1}{14})$ of *r3* appears as a part of the following step of the calculation.

2. $7 \times [(\mathit{\frac{1}{8}} + \mathit{\frac{1}{64}})$ *ḥḳ3t* + $(\frac{1}{2} + \frac{1}{7} + \frac{1}{14})$ *r3*] <= 1 *ḥḳ3t*>

The value which was reached in step 1 is – expressed by means of fractions of the *ḥḳ3t* system – multiplied by 7. Adding of all of the values we get $\frac{63}{64}$ of *ḥḳ3t* + 5 *r3*; that means $\frac{64}{64}$ = 1 *ḥḳ3t*. The test is done and the calculation is finished.

25368 10:	1	7
	10	70
	20	140
	2	12^{sic}
	4	24^{sic}
	$\frac{1}{7}$	1
	$\frac{1}{4}$ $\frac{1}{28}$	2
	$\frac{1}{2}$ $\frac{1}{14}$	4
	\1	$\mathit{\frac{1}{8}}$ $\mathit{\frac{1}{64}}$ [$\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{14}$]
	\2	$\mathit{\frac{1}{4}}$ $\mathit{\frac{1}{32}}$ 1 $\frac{1}{4}$ $\frac{1}{7}$ $\frac{1}{28}$
	\4	$\mathit{\frac{1}{2}}$ $\mathit{\frac{1}{16}}$ 2 $\frac{1}{2}$ $\frac{1}{4}$ [$\frac{1}{14}$ $\frac{1}{28}$]

This copy of the same calculation as was just described follows the steps of the procedure in exactly the same way. However, the scribe also did not manage to avoid making mistakes. The fourth line of the calculation which should have contained the information about the value of 40 × 7 was omitted. Furthermore, the error in the case of the doubling and quadrupling which was mentioned in previous case, also occurs here.

25367 2:	1	7
	10	70
	20	140
	40	280
	2	12^{sic}
	4	24^{sic}
	$\frac{1}{7}$	1
	$\frac{1}{4}\ \frac{1}{28}$	2
	$\frac{1}{2}\ \frac{1}{14}$	4
	1	$\mathit{\frac{1}{8}}\ \mathit{\frac{1}{64}}\ \ \frac{1}{2}\ \frac{1}{7}\ \frac{1}{14}$
	2	$\mathit{\frac{1}{4}}\ \mathit{\frac{1}{32}}\ \ 1\ \frac{1}{4}\ [\frac{1}{7}\ \frac{1}{28}]$

This calculation of a seventh of a *ḥķ3t*, which occurs on the second tablet, was not finished. The first part of the example contains the errors in the double and quadruple lines, the second step consists of only two lines. The reason for this premature ending of the calculation is not evident. However, looking at the position of this example on the tablets, we can suppose that there was not enough room for the last line of the test (see plate I.).

25368 13:	1	[7]
	10	70
	20	140
	40	280
	2	12^{sic}
	4	24^{sic}
	$\frac{1}{7}$	1
	$\frac{1}{2}\ \frac{1}{14}$	2^{sic}
	1	$\mathit{\frac{1}{8}}\ \mathit{\frac{1}{64}}\ \ \frac{1}{2}\ \frac{1}{7}\ \frac{1}{14}$
	2	$\mathit{\frac{1}{4}}\ \mathit{\frac{1}{32}}\ \ 1\ \frac{1}{4}\ \frac{1}{7}\ \frac{1}{28}$
	4	$\mathit{\frac{1}{2}}\ \mathit{\frac{1}{16}}\ \ 2\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{14}\ [\frac{1}{28}]$

This is the last copy of the same calculation. In this case, the scribe made a terrible error during the division. Besides the mistakes in the double and quadruple lines which occur in all four copies of this calculation, the scribe put together the eighth and the ninth lines. There is only one line at their place, which consists of the value $(\frac{1}{2} + \frac{1}{14}) \times 7$ in the left column, but the right column contains the value 2, which corresponds to $(\frac{1}{4} + \frac{1}{28}) \times 7$.

Considering the mistakes which occur in all four copies of the calculation of the seventh of a *ḥḳ3t*, we are entitled to believe that the scribe must have been quite careless. He probably simply copied the calculations and was not attentive enough to notice the mistakes which he had already made in the first case. Which one of the four copies was the first cannot be determined with certainty, but it does not seem to be significant.

The following example is the only calculation which concerns a fraction of a *ḥḳ3t* with an even denominator. The calculation is quite easy and the scribe probably did not need to copy it several times. Thus, it occurs only once on the wooden tablets.

25368 14:	1	10	
	10	100	
	20	200	
	2	20	
	1	$\frac{1}{16}\ \frac{1}{32}$	2
	\2	$\frac{1}{8}\ \frac{1}{16}$	4
	4	$\frac{1}{4}\ \frac{1}{8}\ \frac{1}{64}$	3
	\8	$\frac{1}{2}\ \frac{1}{4}\ \frac{1}{32}\ \frac{1}{64}$	1

1. $320 \div 10 <= 32>$

The first part of the calculation is the division of one *ḥḳ3t*, in form of 320 *r3*, by 10 which is the denominator of the given fraction. The division is quite easy and the result is an integer. As we can see in the first line of step 2, the value 32 *r3* which is the result of the division, was automatically transformed into the *ḥḳ3t* system fractions as $(\frac{1}{16} + \frac{1}{32})$ of *ḥḳ3t* + 2 *r3*.

2. $10 \times [(\frac{1}{16} + \frac{1}{32})$ *ḥḳ3t* + 2 *r3*] <= 1 *ḥḳ3t*>

In this case, the test was very easy too. The correctness of the result was proven by adding the values of its double and its multiple of 8. The sum of the values in the *ḥḳ3t* unit is $\frac{63}{64}$ of a *ḥḳ3t*, the other ones make 5 *r3* which is $\frac{1}{64}$ of a *ḥḳ3t*. The total is 1 *ḥḳ3t* indeed.

The calculation which has to determine the value of an eleventh of a *ḥḳ3t* and to express it by means of the *ḥḳ3t* system fractions, is copied four times on the tablets. As we can see below, not all of them are correct.

25368 4:	1	11
	10	1[10]
	20	2[20]
	2	22
	4	44
	8	88
	$\frac{1}{11}$	1
	\1	$\frac{1}{16}$ $\frac{1}{64}$ 4 $\frac{1}{11}$
	\2	$\frac{1}{8}$ $\frac{1}{32}$ $\frac{1}{64}$ 3 $\frac{1}{6}$ $\frac{1}{66}$
	4	$\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ 1 *sic* $\frac{1}{33}$
	[\8	$\frac{1}{2}$] $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ 2 $\frac{2}{3}$ $\frac{1}{22}$ $\frac{1}{66}$

1. $320 \div 11 <= 29 + \frac{1}{11} >$

The method used to solve this problem is the same as in the previous examples. The first step of the calculation is the division of 320 *r3* by the denominator 11. The division itself is not difficult. After getting the integer part of the result, which is 29 *r3*, the reminder is 1. Thus, an eleventh is to be added to 29 to get the total result of the division. This $29 + \frac{1}{11}$ of *r3* correspond to $(\frac{1}{16} + \frac{1}{64})$ of *ḥḳ3t* + $(4 + \frac{1}{11})$ of *r3*.

2. $11 \times [(\frac{1}{16} + \frac{1}{64})$ *ḥḳ3t* $+ (4 + \frac{1}{11})$ *r3* $] <= 1$ *ḥḳ3t* >

The result which was reached in step 1 is to be multiplied by 11. Thus, the values of 2× result and 8× result are added to it. In the third line of the test, $\frac{1}{3}$ of *r3* is omitted.

If we add the numbers according to the units, we get $\frac{62}{64}$ of a *ḥḳ3t* and 10 *r3*, which is $\frac{2}{64}$. The total, which is one *ḥḳ3t*, thus confirmed the correctness of the result of the division.

25368 8:	1	11
	10	110
	20	220
	2	12 *sic*
	4	24 *sic*
	8	[88]
	[$\frac{1}{11}$	1]
	\1	$\frac{1}{16}$ $\frac{1}{64}$ 4 $\frac{1}{11}$
	\2	$\frac{1}{8}$ $\frac{1}{32}$ $\frac{1}{64}$ 3 $\frac{1}{6}$ $\frac{1}{66}$
	4	$\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ 1 *sic* $\frac{1}{33}$
	[\8	$\frac{1}{2}$] $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ 2 $\frac{2}{3}$ $\frac{1}{22}$ $\frac{1}{66}$

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– ? EXCEEDS CORRECT TOTAL BY 1/16 HK3T

In the second copy of the same calculation, the scribe evidently made an error in step 1. In the lines which correspond to the doubling and quadrupling of 11, there are the values of 12 and 24 at the spot where 22 and 44 should be. The scribe was probably careless and he may have been confused by the similarity between the hieratic signs for the numbers 10 and 20.

Since the rest of the calculation is correct, the scribe seems not to have performed the calculation himself, he probably copied it quite carelessly.

25368 9:	1	11
	10	110
	20	220
	2	22
	4	44
	8	88
	$\frac{1}{11}$	1
	\1	$\frac{1}{16}\ \frac{1}{64}\ 4\ \frac{1}{11}$
	\2	$\frac{1}{8}\ \frac{1}{32}\ \frac{1}{64}\ 3\ \frac{1}{6}\ \frac{1}{66}$
	4	$\frac{1}{4}\ \frac{1}{16}\ \frac{1}{32}\ \frac{1}{64}\ 1^{\textit{sic}}\ \frac{1}{33}$
	\8	$\frac{1}{2}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}\ 2\ \frac{2}{3}\ \frac{1}{22}\ \frac{1}{66}$

This copy of the same example is recorded immediately next to the one described above. Besides the omission of $\frac{1}{3}$ in the third line of the test, it does not contain any mistakes. However, it is difficult to check out whether the scribe realized that there was something wrong in the previous case and this may be why he repeated the calculation once again. He could simply have copied it again just to memorize the procedure.

25368 11:	1	11
	10	110
	20	220
	2	22
	4	44
	8	88
	$[\frac{1}{11}$	$1]$
	\1	$\frac{1}{16}\ \frac{1}{64}\ 4\ \frac{1}{11}$
	\2	$\frac{1}{8}\ \frac{1}{32}\ \frac{1}{64}\ 3\ \frac{1}{6}\ \frac{1}{66}$
	4	$\frac{1}{4}\ \frac{1}{16}\ \frac{1}{32}\ \frac{1}{64}\ 1^{\textit{sic}}\ \frac{1}{33}$
	\8	$\frac{1}{2}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}\ 2\ \frac{2}{3}\ [\frac{1}{22}\ \frac{1}{66}]$

Surprisingly, a fourth copy of the calculation for an eleventh of a *ḥḳ3t* is recorded next to the ones which were just mentioned. This multiple repetition of one procedure in the same place confirms the idea that the scribe intended to practice the calculation procedure. On the other hand, the errors in case number 8, indicate that he sometimes tried to facilitate his work by looking at the pattern.

On the tablets, there are three copies of the calculation which determines a thirteenth of a *ḥḳ3t*. Only one of them is recorded completely, the other examples were ended prematurelly.

25367 3:	[1	13	
	10	130	
	20	260	
	2	26	
	4	52	
	$\frac{1}{13}$	1]	
	$\frac{1}{8}\ \frac{1}{52}\ \frac{1}{104}$	2	
	$\frac{1}{4}\ \frac{1}{26}\ \frac{1}{52}$	4	
	\\$\frac{1}{2}\ \frac{1}{13}\ \frac{1}{26}$	8	
	1	$\mathit{\frac{1}{16}}$ $4\ \frac{1}{2}\ \frac{1}{13}\ \frac{1}{26}$	10
	[2	$\mathit{\frac{1}{8}\ \frac{1}{32}}$ sic] $4\ \frac{1}{13}\ \frac{1}{52}\ \frac{1}{104}$	11
	4	$\mathit{\frac{1}{4}\ \frac{1}{32}\ \frac{1}{64}}$ 2^{sic} $\frac{1}{4}\ \frac{1}{8}\ \frac{1}{13}\ \frac{1}{104}$	12
	8	$\mathit{\frac{1}{2}\ \frac{1}{16}\ \frac{1}{32}\ \frac{1}{64}}$ $1\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{26}\ \frac{1}{104}$	13

1. $320 \div 13 <= 24 + \frac{1}{2} + \frac{1}{13} + \frac{1}{26} >$

The division of 320 *r3* by 13 forms the first nine lines of the calculation. The first lines are hardly visible but they can be restored with the help of other copies of this example which will be discussed below.

2. $13 \times [\frac{1}{16}$ *ḥḳ3t* $+ (4 + \frac{1}{2} + \frac{1}{13} + \frac{1}{26})$ *r3*$] <= 1$ *ḥḳ3t* $>$

The second step which should confirm the result reached in step 1 already operates with the values in the *ḥḳ3t* unit, as it was in the previous cases. There are scribal errors in the twelfth and the thirteenth lines. In the first case, a value of $\frac{1}{32}$ is recorded in the position where $\frac{1}{64}$ should be. In the next line, there is 2 instead of 3 *r3*. However, the errors did not affect the rest of the calculation.

We can check that the sum of the values in the *ḥḳ3t* unit is $\frac{62}{64}$ of a *ḥḳ3t* and the other values make 10 *r3*, that means $\frac{2}{64}$ of a *ḥḳ3t*. After adding both sums, we get $\frac{64}{64}$ which is a whole *ḥḳ3t*.

25367	1:	1	13
		10	130
		20	260
		2	26
		4	52
		$\frac{1}{13}$	1
		$\frac{1}{8}$ $\frac{1}{52}$ $\frac{1}{104}$	2
		$\frac{1}{4}$ $\frac{1}{26}$ $\frac{1}{52}$	4
		$\frac{1}{2}$ $\frac{1}{13}$ $\frac{1}{26}$	8
Ro		1	$\frac{1}{16}$ $\frac{1}{64}$ sic 4 $\frac{1}{2}$ $\frac{1}{13}$ $\frac{1}{26}$
2/13 HK3t = $\boxed{493/13}$		2	40 $\frac{1}{8}$ $\cancel{\frac{1}{32}}$ sic $\frac{51}{64}$ 4 $\boxed{\frac{1}{13}\ \frac{1}{52}\ \frac{1}{104}}$ = 3/13 − 1/8 ! *
		4	$\frac{1}{4}$ $\frac{1}{16}$ sic $\frac{1}{32}$ $\frac{1}{64}$ 3

This is another copy of the same calculation. It was recorded on the same tablet as the example which was just described above. This example is not finished and it contains more mistakes than the first one. However, the beginning of this calculation helps to restore the lines of the previous one, which have almost vanished.

The second part of the calculation, which tests whether the result is correct, was never finished. In its first line, there was an extra $\frac{1}{64}$ of a *ḥḳ3t* added to $\frac{1}{16}$. This error influenced the rest of the procedure of testing. Some redundant values occur in every line. The calculation ended with the quadruple of the result of step 1. In can be supposed that the scribe realized that there was something wrong with the values during the calculation and so he decided to start to perform the calculation again. This new reckoning is the example which was described above as the only complete copy of this calculation.

25368	7:	1	13
		10	130
		2	26
		4	52

We can find a calculation of a thirteenth of a *ḥḳ3t* also on the second tablet. In this case, the procedure was ended after a few lines. This may be related to the fact that the line with the value of 20 × 13 was omitted.

Summary

The calculations recorded on the Cairo wooden tablets can be described as transformations of certain fractions into the system of the *ḥḳ3t* unit, the so-called "fractions of the eye of Horus".

Two different methods can be distinguished on these tablets. One of them was used in determining $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{13}$ of a *ḥk3t*. In this case, a simple division was performed. A *ḥk3t* which had to be divided was expressed as 320 *r3*. After the division, a test was made to prove that the result was correct.

Another method was preferred for determining $\frac{1}{3}$ of a *ḥk3t*. First, a third of 5 *r3* was reckoned, that means a third of $\frac{1}{64}$ of a *ḥk3t* which is the smallest fraction of the *ḥk3t* system. By doubling this value, the values were reached which correspond to all of the fractions of the eye of Horus system. The total result was then easily obtained. A test was performed at the end of the calculation to show that the result was correct.

Concerning the use of a different method in the case of $\frac{1}{3}$ of a *ḥk3t*, already T. E. Peet has remarked that the ancient Egyptians must have been perfectly familiar with calculations involving $\frac{1}{3}$ and $\frac{2}{3}$, since these two fractions played such an important role in Egyptian mathematics[10]. Thus, it seems ingenious that they preferred a method which was based on the use of these two fractions. Furthermore, the fact that the system of the "fractions of the eye of Horus" operated with values which could be obtained from each other by doubling (or dividing by 2) was advantageously utilized in this calculation.

The same procedure could have been used in cases which were mentioned in detail in the body of this essay. However, it seems that the ancient Egyptians thought that this method would not be so useful in those particular examples. Thus, the advantage of reckoning with $\frac{1}{3}$ and $\frac{2}{3}$ seems to have been essential for using two different methods for determining a fraction of a *ḥk3t*.

The choice concerning the different methods was probably not made by the scribe who recorded the calculations on the Cairo tablets himself. Undoubtedly, the procedures were already well established at that time. The methods of such transformations must have been created already in the period when the *ḥk3t* unit and its system of fractions came into use. Then, the *ḥk3t*-pioneers probably created fitting procedures for different fractions of a *ḥk3t*.

Conclusions

It can be supposed that in the calculations which are recorded on the Cairo tablets, the procedures were much more important than the results – otherwise the scribe could have used tables of values. Such tables are, for example, known for the addition of fractions (leather roll)[11] or for determining of an odd fraction of 2 (the so-called $2 \div n$ tablets in the Rhind papyrus and Kahun papyri)[12].

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[10] Peet, T. E.: "Arithmetics in the Middle Kingdom", *JEA* 9, P. 95.

[11] Glanville, S. R. K.: "The Mathematical Leather Roll in the British Museum", *JEA* 13 (1927), pp. 232-239.

[12] Rhind papyrus: Eisenlohr, A.: *Ein mathematisches Handbuch der alten Aegypter*, Leipzig 1877; Peet, T. E.: *The Rhind Mathematical Papyrus*, London 1923; Chace, A. B.: *The Rhind Mathematical Papyrus*, Oberlin, Ohio 1979; Robins, G. – Shute, Ch.: *The Rhind Mathematical Papyrus*, London 1990; Kahun fragments: Griffith, F. L.: *Hieratic Papyri from Kahun and*

In this case, however, the result itself does not seem to be that important, since it is not highlighted as the results of mathematical problems usually are[13]. The scribe also seems to have been quite familiar with the *ḥḳ3t* system itself, since the transformations of values between units *r3* and *ḥḳ3t* are performed quite automatically. Thus, we have to emphasize once more that the procedures and not the results were important here.

If we analyze the methods used in the calculations, we can see that the scribe performed only the division and then the multiplication of fractions. The operations with fractions formed a very important part of Egyptian mathematics, but the core of the problem could have been something else – not only the division itself but the fact that two types of fractions were employed in the calculations, the "ordinary" fractions and the "fractions of the eye of Horus" for which different hieratic signs were used.

It is evident that the scribe felt he had to emphasize the working out of the procedures. He systematically exercised *how* to get the results of such problems, so that he would be able to reckon the value of any given fraction of a *ḥḳ3t*. This does not prove that he had to understand it; on the other hand, it indicates that Egyptian mathematics was not based mainly on memorizing as it is sometimes considered to be.

The Cairo tablets and the calculations of fractions of a *ḥḳ3t* connect Egyptian mathematics with the practical experience of scribes. In mathematical papyri, such as the Rhind papyrus, Moscow papyrus and fragments from Kahun and Berlin, we can find examples concerning teaching and learning the basic knowledge of mathematics in ancient Egypt; the significance of the Cairo tablets resides in the fact that they show its application.

Gurob, London 1898, plate 8, no. IV.2; also see: Vymazalová, H.: *Řešení matematických problémů v egyptských textech* [Methods of solutions of mathematical problems in the Egyptian texts], Praha 2001, M.A. thesis, pp. 43-53.

[13] On the problems of the Rhind papyrus, see footnote 12. Moscow papyrus: Struve, W. W.: *Mathematischer Papyrus des Staatlichen Museums der schönen Künste in Moscau*, Berlin 1930; Kahun fragments: see footnote 12; Berlin fragments: Schack-Schackenburg, H.: "Der Berliner Papyrus 6619", *ZÄS* 38 (1900), pp. 135-140; Schack-Schackenburg, H.: "Das kleinere Fragment des Berliner Papyrus 6619", *ZÄS* 40 (1902), pp. 65-66.

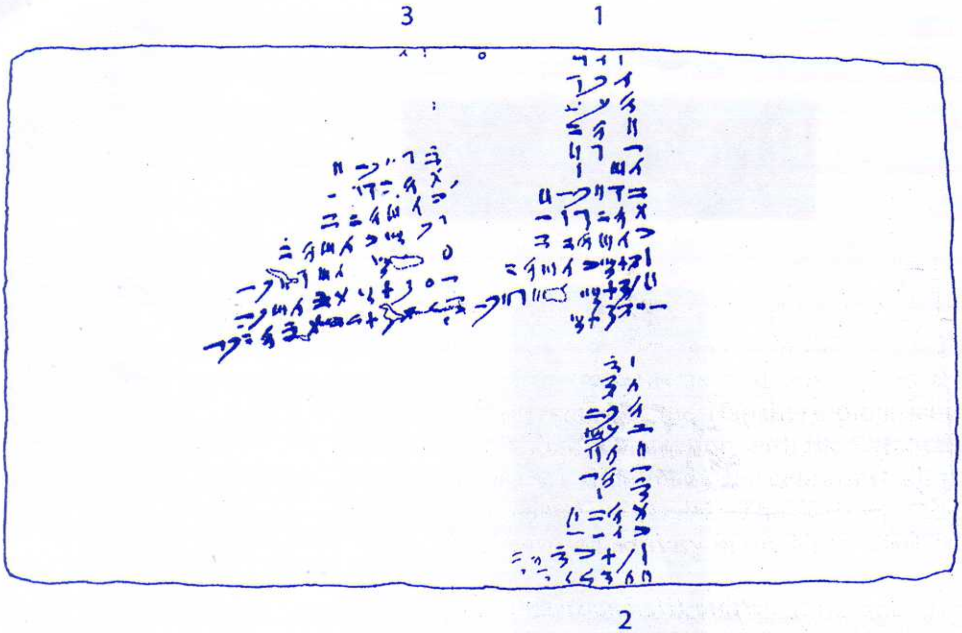

Plate I: The calculations on the tablet 25367.

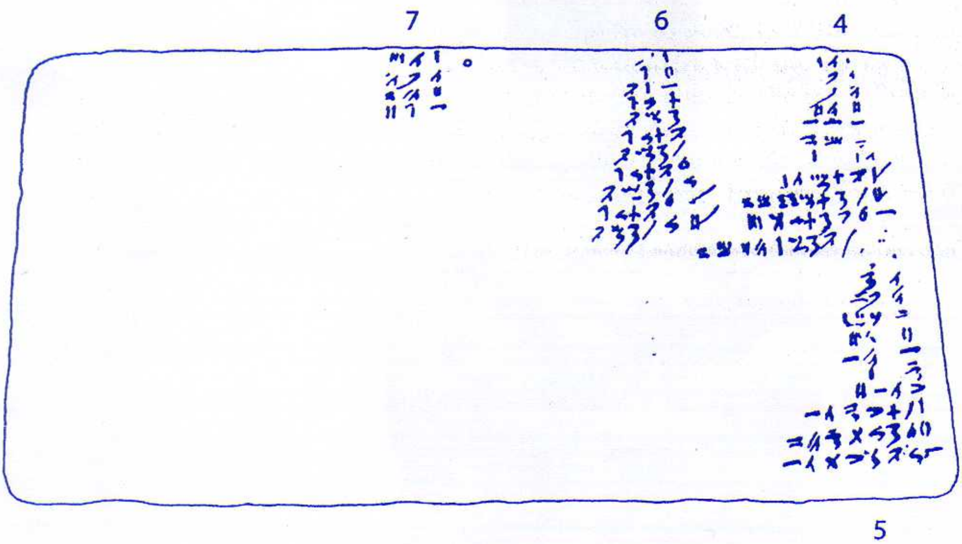

Plate II: The calculations on the tablet 25368.
The left side of the tablet which contains the list of names was left empty here.

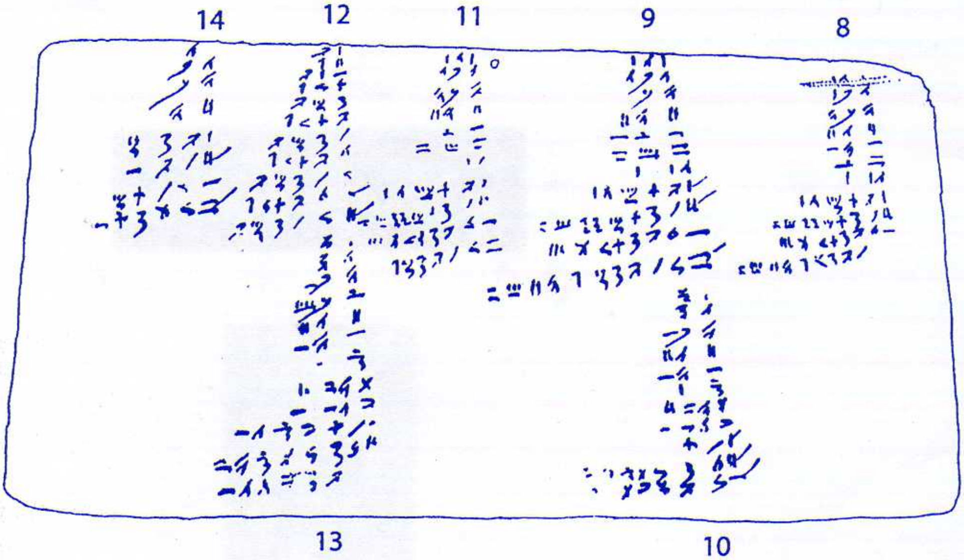

Plate III: The second side of the tablet 25368.