AFRICAN MATHEMATICAL UNION

COMMISSION ON THE HISTORY OF MATHEMATICS IN AFRICA

AMUCHMA-NEWSLETTER-15

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1. OBJECTIVES

The A.M.U. Commission on the History of Mathematics in Africa (AMUCHMA), formed in 1986, has the following objectives:

a. to improve communication among those interested in the history of mathematics in Africa;

b. to promote active cooperation between historians, mathematicians, archaeologists, ethnographers, sociologists, etc., doing research in, or related to, the history of mathematics in Africa;

c. to promote research in the history of mathematics in Africa, and the publication of its results, in order to contribute to the demystification of the still-dominant Eurocentric bias in the historiography of mathematics;

d. to cooperate with any and all organizations pursuing similar objectives.

The main activities of AMUCHMA are as follows:

a. publication of a newsletter;

b. setting up of a documentation centre;

c. organization of lectures on the history of mathematics at national, regional, continental and international congresses and conferences.

2. ON MATHEMATICAL ACTIVITIES IN NORTH AFRICA SINCE THE 9TH CENTURY

FIRST PART: MATHEMATICS IN THE MEDIEVAL MAGHREB

INTRODUCTION

The text that we will present here is the first part of a study which attempts to present the essential aspects of mathematical activity in North Africa since the 9th century. The abundance of material has obliged us to report, in a future issue of the Newsletter, the study dedicated to mathematical activities in Egypt during the same period. The whole work, as soon as concluded, will constitute the second volume of the study "*Recent research on the history of Mathematics in Africa: an overview*", by A. Djebbar and P. Gerdes, whose first volume has already been published [GERDES 1992: 3-32, 1994].
This work is just a first outline that needs to be completed by the presentation of certain activities which abundantly used the different mathematical disciplines and were sometimes considered more important than mathematics.

Essentially this refers to Astronomy, Astrology, and the Science of Succession Division or Science of Inheritance. We have satisfied ourselves by evoking certain of their aspects, or certain authors who are particularly distinguished, hoping to deal with them more fully in a later study.

Before everything else, and to avoid eventual misunderstandings or ambiguities, it seems useful to make some remarks about the terminology that will be used in the different parts of this study.

First of all we need to specify what we understand by "*scientific tradition*". As soon as we begin to deal with the contents of scientific, and more particularly mathematical activities, which take place in the context of the Arab-Islamic civilisation, it is not possible to speak of a specific tradition in the Maghreb (as opposed to that of Moslem Spain or that of the East). In fact we may speak of one overall tradition, that of Arabic mathematics — that is mathematics that has been thought about, written and taught in the Arabic language (also called Mathematics of the countries of Islam) —, which has developed in the East since the end of the 8th century, and that has been partially transmitted to the cities of the Moslem West and of Central Asia, and later to southern Europe, by means of translations (essentially Latin and Hebrew). This tradition has been assimilated, revived and enriched by the scientific environments of the different countries of Islam that gave it, sometimes, certain specific mark at the level of this or that research and teaching orientation, as well as at the level of composition of the contents of the works, of the terminology, or of the classification of the studied disciplines. But, as far as we know, this internal process of differentiation has not lead to the emergence, at local or regional level, of a new mathematical tradition characterised by its own concepts and paradigms. The scientific traditions which are evoked in this study are such in an externalist sense that takes scientific practice in relation to its environment into account.

The same remark applies to the contents of the mathematics produced or taught in each of the five vast regions that constituted, in the Middle Ages, North Africa, that means the Extreme Maghreb, the Central Maghreb, the Oriental Maghreb that at that time was called *Ifrīqyā*., Egypt and the extended sub-Saharan zone of Moslem confession that went then by the name of *Bilād as-Sūdān* [Countries of the Sudan]. The analysis of the scientific texts which have come to us does not allow us to speak of regional specificities either at the level of content itself or at the evolution of its contents. On the contrary, we observe, following the epochs, notable differences between the regions, at the level of the number of mathematicians or of their works, of the vitality of this or that taught discipline, of the dynamism of the different scientific foyers of each of these regions.

It is also necessary to explain what we here understand by "*Maghrebian mathematicians*". In bio-bibliographic treatises, in particular those written by oriental authors, one finds a certain number of scholars, poets and writers who are referred to as "*maghrebian*" without being born in the Maghreb. That is the case, for example, of scholars native to Moslem Spain. That is also the case

of those whose parents are native of the Moslem West but who have grown up or have been educated in the East. In addition, there are persons whose origin is not Maghrebian but who have played a role in the scientific activity of the Maghreb. Thus, in the case of mathematicians, it is necessary to specify that only one category is representative of the mathematical production of the Maghreb. This concerns those who lived there during a given period of their life and who, through their teaching or through their production, contributed to the development or the perpetuation of a local or regional mathematical activity. As an example of this type of scholar, one may cite al-Qat;. rawānī (15th c.), who was born in Egypt and who lived for a certain time in Tunis where he wrote one of the books that we will mention later [LAMRABET 1981: 41-91; DJEBBAR 1986a: 118-19; HADFI 1989, 1992: 138-39].

With respect to the others, one may classify them into two categories: the first regroups all who were natives of the Maghreb and who have left it in order to install themselves in another region of Africa (such as Egypt or the Sub-Saharan regions of the continent). Among the representatives of this category, one finds al-H;. asan al-Murrākushī, one of the greatest specialists of Astronomy in the 13th century, who lived essentially in Egypt [SEDILLOT 1834-35, 1844; SOUISSI 1982; MURRĀKUSHĪ 1984].

The second category regroups all those who were born in or were natives of the Maghreb but who were, in reality, educated in the cities of the East or of Moslem Spain, and who spent the largest part of their life there. One of the most eminent representatives of this category is as-Samaw'al al-Maghribī (d. 1175) who was born in Baghdad to a Jewish family native of the Extreme Maghreb, more precisely from Fez [ANBOUBA 1970: 80]. His major contributions, which concern the theory of polynomials and decimal fractions, seem not to have been known by the Maghrebian mathematicians of the 12th-14th centuries [RASHED & AHMAD 1972; RASHED 1984].

As these mathematicians or astronomers have not participated, in any way, in a scientific activity in the Maghreb and as these have not been vectors of this activity, we have not judged it useful to mention their different contributions here.

BIRTH AND FIRST DEVELOPMENTS OF MATHEMATICAL ACTIVITIES IN THE MAGHREB (9TH-11TH C.)

Taking into account the very close economical, political and cultural links which were woven between the Maghreb and Moslem Spain throughout the Middle Ages, and taking into account the quantitative and qualitative importance of the transmission of the scientific production of each of these regions to the other, it seemed necessary for us to briefly recall the genesis and the most important aspects of the development of mathematical activity in the cities of Moslem Spain. This could also help the reader to better appreciate the strong lines of orientation of mathematical activity in the Maghreb by placing

them into a larger — but more natural for the epoch — cultural and scientific context, that is that of the Moslem West.

In fact, the period that extends from the end of the 8th century to the end of the 11th century is characterised by the development, in the Maghreb and in Moslem Spain, of two more-or-less linked scientific traditions encouraged by scholars who, beyond the social contradictions and the differences of statute or of religion, were relatively united both by the way of life of the Islamic city and by the cultural and scientific environment that had been established favouring different human contributions and multiple contacts with the scientific foyers of the Moslem East [VERNET 1978; VERNET & SAMSO 1981; SAMSO 1992].

Having said this, we have to note immediately that the birth and the first steps of scientific activities in Moslem Spain and in the Maghreb are not well known. In fact, and to limit ourselves to our discipline, one has to recognise that testimonies concerning the beginning of mathematical activities in these two regions of the Moslem West are rare and not very specific. Speaking of the scientific activities during the period that follows the Moslem conquests in Spain, the bio-bibliographer of the 11th century, S;. āᶜid al-Andalusī, says that "the country remains indifferent to all sciences, with the exception of those of Law and the Arabic language, until the day that the power passes definitely to the Umayyads", that is around the middle of the 8th century [S;. ĀᶜID 1987 : 120]. For his part, Ibn Juljul, another Andalusian bio-bibliographer who lived in the 10th century, leaves us to understand that, until the epoch of the fourth Omeyyad caliph, ᶜAbd ar-Rah;. mān II (826-852), the medical, philosophical and mathematical sciences did not have eminent representatives. But, in saying so, this author confirms implicitly the existence of men of science, understanding maybe that they had still not achieved the level of their colleagues of the East [IBN JULJUL 1955: 76].

However that may be, it seems reasonable to us to think that, during the period of installation and consolidation of Moslem power in the first cities of Spain and of the Maghreb, that Medicine and Calculation were the first scientific disciplines to have benefited from teaching followed by the publication of works, and this to respond to the needs of certain well-to-do ranks of society of the cities, or to the solicitations of lawyers for the solution of certain problems such as those involved in land measurement or in the partitioning of inheritance.

The Andalusian tradition

For what concerns Moslem Spain, it seems that since the beginning of the 9th century, the children of princes, of dignitaries or of well-to-do persons, have benefitted from scientific teaching using the first copies of translations of Greek and Indian works, made in the foyers of the centre of the empire, and maybe even copies of the first Arabic teaching books that started to appear in Baghdad, from the end of the 8th century. This might have been the case of the

children of the merchant and royal families, in particular those of ᶜAbd ar-Rah;. mān II [IBN SAᶜīD 1978: I, 45]. One knows also that the last one, after becoming caliph, participated in his turn in the support and the dynamisation of scientific activities by financing the establishment of an important library, and by providing it with books bought in the East. These caliphal initiatives, and probably other private initiatives, of which no precise testimonies have come to us, could only have favoured the quantitative and qualitative development of scientific activities in the principal cities of Moslem Spain. However that may be, it seems that it is in this period, that is around the middle of the 11th century, that consequent scientific foyers started to exist on their own, in Cordoba and in other cities such as Toledo, Seville, Zaragoza and Valencia, which already knew relative economic prosperity [S;. ĀᶜID 1987: 122]. It is also in this period that one meets the first scholars whose names were transmitted by bio-bibliographers, together sometimes with some information on their profile or on their activities [IBN BUSHKUWĀL 1966; AD;. -D;. ABBī 1884; IBN AL-ABBĀR 1886].

During the last third of the 9th century and throughout the 10th, teaching and research, in the different fields of mathematics, were given greater importance due to the patronage of the two great Omeyyad caliphs of the 10th century, ᶜAbd ar-Rah;. mān III (912-961) and his son al-H;. akam II (961-976). One sees thus a real research tradition being established around high level professors like Maslama al-Majrīt;. ī (d. 1007), who wrote works of mathematics and astronomy rivalling those that were produced in the East in the same epoch, and one sees a greater and greater number of young researchers emerge, like Ibn as-Samh;. and az-Zahrāwī, who dominated the scientific activities of the first half of the 11th century and whose books were an authority both in Spain and in the Maghreb [SEZGIN 1974: 334-35, 355-56].

At the internal level of scientific tradition, one does not always have direct and precise testimonies on the nature and content of the exchanges that took place, during this period, between, on the one hand, the East and the West, and, on the other hand, between Moslem Spain and the Maghreb. But the analysis of the mathematical texts that came to us allows us to say that students, teachers and researchers had at their disposal initially translations of fundamental Greek texts, such as Euclid's *Elements*, Ptolemy's *Almagest*, Apollonius' *Conic Sections*, and Archimedes' *On the Sphere and the Cylinder*, and other works of less importance but essential to the training of a future mathematician or a future astronomer, such as Euclid's *Data*, Archimedes' *Lemmas*, and Menelaus' *Sphaerica*. Later (or maybe simultaneously in the case of Algebra), one studied certain works of Arabic scholars from the East, like the *Book on Indian calculation* and the *Book of Algebra* by al-Khwārizmī (d. 850), the *Treatise of the secant figure* and the *Treatise on amicable numbers* by Thābit Ibn Qurra (d. 901), the book of Banū Mūsā (9th c.) on *The measure of plane and spherical figures*, the *Book of Algebra* of the Egyptian Abū Kāmil (d.

930), as well as other writings of the same epoch, like the *Epistle on proportion and proportionality* and *Epistle on similar arcs* by Aḥmad Ibn ad-Dāya (d. 944), another mathematician from Egypt [SEZGIN 1974: 288-90].

The Maghrebian tradition

Concerning the Maghreb we may say that the testimonies which have come to us on scientific activities between the 9th and the 11th century allow us to think that the beginning of mathematics, in this region of North Africa, took place in Ifriqiya and more precisely at Kairouan, from the end of the 8th century and that these activities have remained confined within the limits of this region for a long time. Some names of scholars have come to us, like that of Yaḥyā al-Kharrāz [IBN TAMĪM: 90-91] and that of his pupil Yaḥyā al-Kīnānī (828-901), author of the first Maghrebian book of Ḥisba (which deals with the rules of commercial transactions at the market places) [IBN AL-FARAḌĪ 1966: II, 183]. Maghrebian sources cite also Shuqrān Ibn ᶜAli who was a specialist in Calculation and in the Science of Inheritance and who is perhaps the first Maghrebian to have written a book on the partitioning of successions [MAKHLŪF 1930: n° 31]. According to the testimony of Ibn Khayr (12th c.), the content of this book was still taught in the 12th century, at Bougie, a scientific metropolis of the Central Maghreb [ZERROUKI 1995: 15].

For the 9th century, the name of only one mathematician has survived. It is that of Abū Sahl al-Qayrawānī, whose parents were natives of Baghdad. He is also the first known Maghrebian mathematician of whom the title of one of his treatises has come to us. It is called *Kitāb fi l-ḥisāb al-hindī* [Book on Indian calculation]. As its title clearly indicates, this book belongs to the new Arabic arithmetical tradition, of Indian origin, which started at the end of the 8th or at the beginning of the 9th century, in the handbooks of the mathematicians of the East.

It seems that it was the eminent role played by Kairouan in the theological debates, at the Aghlabid epoch (800-910), that attracted numerous intellectuals from the East to Ifriqiya, such as Abū Sahl and, among them, of men of science educated in arithmetical and geometrical techniques that could serve, in particular, to solve problems of land measurement or inheritance.

As in the other regions of the countries of Islam, the patronage in favour of scientific activities existed in the Maghreb between the 9th and the 11th century, and functioned in the image of that of the great metropolis of the East: buying of books, financing of copies of manuscripts, remuneration of scholars, construction of schools or institutions. This is not surprising if one knows the links which are woven between the caliphate of Baghdad and the Aghlabid dynasty that had governed Ifriqiya until the beginning of the 10th century, and when one knows that the kings of this dynasty imitated both the model of government of the caliphs and their initiatives in favour of science and culture in general.

However the only information we have, with respect to this patronage, concerns the *Bayt al-h;. ikma* [House of Wisdom] founded by Ibrāhīm II (875-902) and that bears indeed the same name as of that famous institution created by the Abbassid caliph Hārūn ar-Rashīd(786-809) who has played an great role in the phenomenon of translation of Greek and Indian scientific works [ABDALWAHHAB 1956: 253-72]. This institution, which survived its founder as a scientific centre until the arrival of the Fatimid dynasty, would have received mathematicians, astronomers and astrologers, such as at;. -T;. allā' [ZUBAYDĪ 1954: 164] and ᶜUthmān as;. -S;. ayqal [ABDALWAHHAB 1965-72: I, 249-50, 252-54].

The mathematical activities in the Maghreb, during the 10th century are very little known. It seems that the patronage started by the Aghlabids, in the 9th century, was continued and profited Mathematics and Astronomy, in particular in the course of the first two decades of the government of the Fatimid caliph al-Muᶜizz (953-975) [NUᶜMĀN: 91]. However, nothing has come to us in the form of scientific documents which might inform us about the content of that what was produced or taught at that time. The biographers have only retained some names of persons who made themselves known by their activity in mathematics or by their interest in this discipline. As examples, one may cite al-ᶜUtaqī al-Ifrīqī (d. 955) [SUTER 1900: 70-71], Yaᶜqūb Ibn Killīs (d. 990) and al-Huwarī (d. 1023) [ZIRIKLĪ 1980: VIII, 202-3, 158-59].

We are relatively better informed on the mathematical activities of the 11th century. But our knowledge remains still very fragmentary. Certain scholars of this period are better known. This is the case of Ibn Abi r-Rijāl (d. 1034-35) who published works in Mathematics and Astronomy, which have not come down to us, and who was equally interested in Astrology. It is indeed due to this last discipline that he has been known in Europe since the 12th century, as his book *al-Bāriᶜ fī ah;. kām an-nujūm* [The brilliant book on the judgments of the stars] was translated by Constantin the African [BOUYAHYA 1972: 83-88]. This is also the case of Abū s;. -S;. alt (d. 1134), a native Spanish scholar, who spent most of his life in Egypt and then in Ifriqiya, and who published works concerning Geometry, Astronomy and Logic [DJEBBAR 1988b: 61-66]. Other mathematicians are less well known, such as ᶜAbd al-Munᶜim al-Kindī (d. 1043-44) and Ibn ᶜAt;. iya al-Kātib (c.a. 1016) [DABBAGH 1902: III, 228; BOUYAHYA 1972: 146]. We know that they occupied themselves with Geometry and Arithmetic, but we are still ignorant of their links with the different scientific foyers of their epoch and, in particular, about the circulation and the impact of their mathematical writings in the cities of the Maghreb .

MATHEMATICS IN THE MAGHREB DURING THE ALMOHAD EPOCH (12TH-13TH C.)

The importance of the 12th century in the political and economic history of the Maghreb has often been underlined by the specialists. It is thus reasonable to think that this importance extends to other spheres of social life. However, the cultural and scientific history of this period still remains, although some contributions exist, a vast unexplored field. To take the example of mathematics, one is obliged to note that, in the light of the research results, if one disregards two or three little algebraic or arithmetical poems that we will evoke later, only four works (of which one is incomplete) have come to us. It needs, however, to be specified that the discovery and the analysis of some of them took place only a few years ago.

The authors of these works are, in chronological order, al-H;. as;.s;. ār (12th c.), Ibn al-Yāsamīn (d. 1204) and Ibn Munᶜim (d. 1228), to which one should add a fourth, al-Qurashī (d. 1184), who had as much influence as the others through his writings, which unfortunately have not come to us but on the contents of which we have some information.

The importance of these four scholars is due to several reasons : in the first place, and independently of their origins, they may be considered mathematicians of the Maghreb in the sense that all four seem to have lived there and published certain of their mathematical works, even if some of them were educated, totally or partially, in a city of Moslem Spain. In the second place, these are the first mathematicians of this part of North Africa of whom the contents of certain of their writings have come to us, permitting us thus to have direct information on important aspects of mathematical activity in this region. In the third place, one may consider the works written in this epoch as direct testimonies of two closely linked phenomena. The first is the diffusion, at a larger scale than before, of part of the Andalusian mathematical tradition towards North Africa, through the intermediary of the cities of the Maghreb, such as Sebta, Fez and Marrakech for the Extreme Maghreb, Bougie for the Central Maghreb, and Tunis for Ifriqiya. The second phenomenon is the redynamisation of the teaching and research activity in the Maghreb, in particular due to the patronage of the first Almohad caliphs and then of those dynasties who have divided their empire and who have made efforts to continue their patronage in favour of theologians, poets, writers and scholars.

The contribution of al-Qurashī

We know little about the life of Abu l-Qāsim al-Qurashī, his education and his scientific production. This scholar, who was a native of Seville, in Spain, spent a period of his life in Bougie where he died in 1184. The biographers who evoked him consider him a specialist in Algebra and in the Science of Inheritance (next to his specialisation in certain religious sciences) [ZERROUKI 1995: n° 5, 10-19].

In Algebra, al-Qurashī is known for his commentary on the book of the great Egyptian mathematician Abū Kāmil (d. 930). This commentary has not yet been recovered but its importance is confirmed by the historian Ibn Khaldūn

(d. 1406) who considered it one of the best treaties written on the book of Abū Kāmil [IBN KHALDŪN 1967: 899], and this is the judgment of somebody who knows because Ibn Khaldūn had an excellent mathematical education, and one even attributes him a youth of writing on this discipline [DJEBBAR 1988b: 68-69]. Relative to the content of the treatise of al-Qurashī, we have found interesting fragments in the book of Ibn Zakariyā' (d. 1404). These few elements permit us to say that this work of al-Qurashī was not a simple commentary on a famous treatise of algebra of its time. One finds here indeed some new elements, first of all, at the level of presentation, since al-Qurashī starts by presenting the objects and the operations of Algebra before explaining the solution of the canonical equations followed by the demonstration of the existence of the solutions of these equations. In these two chapters, one remarks that al-Qurashī distinguishes himself from his predecessors in the classification of the six canonical equations and in the demonstrations [DJEBBAR 1980: 8-10, 1986a: 106-7]. This work continued to be studied and taught in the Maghreb until the 14th century, as confirmed by Ibn Zakariyā'. One may even suppose that it is the importance of this book that led Ibn al-Bannā (d. 1321) to write, some decades later, his *Kitāb al-us;. ūl wa l-muqaddimāt fi l-jabr* [Book of the foundations and of the preliminaries in Algebra] [DJEBBAR 1990c].

In the domain of inheritance, al-Qurashī is known for having elaborated a new method based on the decomposition of the numbers in prime factors in order to reduce the fractions that intervene in the distribution of a given inheritance to the same denominator [ZERROUKI 1995: n° 6]. His method was very quickly appreciated by the mathematicians who wrote the handbooks explaining it and by showing its usefulness through the presentation of concrete problems of inheritance. Among these mathematicians, one may cite al-ᶜUqbānī (d. 1408) and al-Qalas;. ādī (d. 1486) [HARBILI 1996: n° 7]. Having said this, and in spite of the effectiveness of the method of al-Qurashī, the majority of the practitioners of Law continued, and still today continue, to utilise the ancient method [LAABID 1990].

The contribution of al-H;. as;.s;. ār

Until today we have not found either any biography of al-H;. as;.s;. ār or any precise element concerning his origin and his dates of birth and death. The only information at our disposal, and that are given either by Ibn Khaldūn or by later Maghrebian mathematicians exclusively concern his mathematical production. It seems that this mathematician was equally well-known as a reader of the Koran and as an inheritance specialist, and that he had a high rank as he bore the title of Shaykh al-jamāᶜa [Chief of the Community]. One may equally suppose that he lived for some time, or that he exercised his activities, at Sebta (in the Extreme Maghreb), taking into account the links he seems to have had with the mathematicians of this city.

Two writings of al-H;. as;.s;. ār have survived. The first, entitled *Kitāb al-bayān wa t-tadhkār* [Book of proof and recall] is a handbook of calculation treating numeration, arithmetical operations on whole numbers and on fractions, extraction of the exact or approximate square root of a whole of fractionary number and summation of progressions of whole numbers (natural, even or odd), and of their squares and cubes. Notwithstanding its classical content in relation to the Arab mathematical tradition, this book occupies a certain important place in the history of the mathematics in North Africa and this for three reasons: in the first place, and notwithstanding the development of research, this manual remains the most ancient work of calculation representing simultaneously the tradition of the Maghreb and that of Moslem Spain. In the second place, this book is the first wherein one has found a symbolic writing of fractions, which utilises the horizontal bar and the dust ciphers i.e. the ancestors of the digits that we use today (and which are, for certain among them, almost identical to ours) [WOEPCKE 1858-59: 264-75; ZOUBEIDI 1996]. It seems as a matter of fact that the utilisation of the fraction bar was very quickly generalised in the mathematical teaching in the Maghreb, what could explain that Fibonacci (d. after 1240) it has used in his *Liber Abbaci*, without making any particular remark about it [DJEBBAR 1980 : 97-99; VOGEL 1970-80]. Thirdly, this handbook is the only Maghrebian work of calculation known to have circulated in the scientific foyers of south Europe, as Moses Ibn Tibbon realised, in 1271, a Hebrew translation of it.

The second work of al-H;. as;.s;. ār is entitled *al-Kitāb al-kāmil fi s;. inācat al-cadad* [The complete book on the art of number]. Only the first part of this work, that consists of 117 folios, was recovered and identified in 1986 [ABALLAGH & DJEBBAR 1987: 147-58]. Its content return to themes of part of the first book relative to whole numbers, by developing them, and presents new chapters like one on the decomposition of a number in prime factors, one on the common divisors and multiples and also one on the extraction of the exact cubic root of a whole number. The second part of the work, that has not yet been recovered, but of which we have been able to reconstitute a part of the chapter titles, is dedicated to operations on fractions, to the summation of the different categories of whole numbers and to the exposition of the algorithms that allow for the calculation of perfect, deficient, abundant, and amicable numbers.

The natural question that has to be posed in relation to these two books concerns their link with a possible regional mathematical tradition, and also about the nature of these links. Unfortunately, and until today, no work of Calculation produced in Spain or in the Maghreb before the 12th century, has come down to us. But the reference of al-H;. as;.s;. ār to two Andalusian works of the 11th century, lost today, the *Kitāb al-mucāmalāt* [Book of the transactions] of az-Zahrāwi and the *Mudkhal al-camali* [Practical introduction] of Ibn as-Samh;. , allows us to say that, in one way or another, the arithmetical tradition of Moslem Spain was present in the Maghreb during the 12th

century. This presence will later be reinforced, both by the direct diffusion of Andalusian works treating Algebra, Geometry and Astronomy, and by the utilisation of the contents of the works of al-H;. as;.s;. ār by the later Maghrebian mathematicians. This is what Ibn Munᶜim and Ibn Ghāzī (d. 1513) will do, for example, who cite explicitly the passages of one or the other of the two books, or who refer to them specifically [ABALLAGH & DJEBBAR 1987: 152-53].

The contribution of Ibn al-Yāsamīn

The third representative of the Magrebian mathematical tradition of the 12th century is relatively better known than the two preceding ones. According to his biographers, his mother, whose first name was Yāsamīn [Flower of jasmine], was black (a colour that he inherited) and his father was a native of the Banū l-Ḥ;. ajjāj Berber tribe [IBN SAʿĪD 1945: 42]. One knows also that he lived for a time in Seville where he probably perfected himself in Mathematics, before returning to the Maghreb and installing himself in Marrakech which was then the capital of the Almohad empire. For a long time, this mathematician was only known due to a minor work of 52 lines, his *Urjūza fi l-jabr* [Poem on Algebra] [DJEBBAR 1986a]. It is very possible that it was the success of this poem that incited him to write another one of the same type, that deals with the roots of numbers, and perhaps even a third one that summarizes the method of false position and of which a copy is attributed to him [SHAWQI 1987; ZEMOULI 1993; KOUIDRI 1996]. But the bio-bibliographical sources do not say anything about these last two poems.

These same sources are equally silent regarding a work of the same author, entitled *Talqiḥ;. al-afkār bi rushūm ḥ;. urūf al-ghubār* [Fecundation of the spirits with the symbols of the dust ciphers]. This work is much more important than the three poems, both at the quantitative and the qualitative level. In fact, we are dealing with a book of more than 200 folios that deals equally with the classical chapters of the Science of Calculation and certain chapters of Geometry relative to the calculation of areas. It is indeed, among the works of the Moslem West that have come down to us, the only one that links these two subjects. Its importance is equally due to the nature of his materials and mathematical instruments which make it an original book and certainly also significant to this period of transition where three mathematical practices run in parallel before flowing together into the same stream: that of the East, of Moslem Spain and of the Maghreb [ZEMOULI 1993].

As an example, one may indicate the following elements that contribute to the originality of the work and to its anchorage in the great Arabic mathematical tradition of the 9th-11th centuries: in Arithmetic, and contrarily to the Magrebian tradition that will continue from the 14th century, Ibn al-Yāsamīn treats first multiplication and division before addition and subtraction. This procedure that one will find later with Ibn Zakariyā', seems to have its origin in the mathematical practice of Moslem Spain [GUERGOUR 1992].

In the domain of fractions, the remarks and suggestions of Ibn al-Yāsamīn, relative to the reading of certain expressions, show that, besides the fraction bar, the rest of the symbolism was not yet definitively fixed at his epoch. This book is also the oldest, to our knowledge, that contains both the objects and the operations of Algebra which permit the writing and solution of equations or abstract manipulation of polynomials. One observes indeed two things regarding this symbolism: first the silence of Ibn al-Yāsamīn about this practice, as if he used instruments introduced by other mathematicians before

him, and further the total resemblance of the symbols intervening in his book with those that one finds in the works of the 14th-15th centuries, as those of Ibn Qunfudh (d. 1407), of al-Qalasādī (d. 1486) or of Ibn Ghāzī. If this fact was confirmed by other testimonies, one would then be in the presence of a much older symbolic practice than one believed [WOEPCKE 1854b; DJEBBAR 1980: 41-54; GUERGOUR 1990; ZEMOULI 1993].

Regarding the presence of Geometry in a work on the Science of Calculation, this does not constitute an exceptional fact with regard to the Arab mathematical tradition taken as a whole, in the measure where similar chapters (i.e. treating problems of metric geometry) have already been included in works edited in the East, such as *at-Takmila fi l-ḥ;. isāb* [The complement in calculation] of al-Baghdādī (d. 1037) [SAIDAN 1985] or *al-Kitāb al-Kāfi* [The sufficient book] of al-Karajī (d. 1029) [CHELHOUB 1989].

Notwithstanding this richness of the *Talqīḥ*;. of Ibn al-Yāsamīn, we have not found any explicit reference to this book in later Maghrebian mathematical writings. This might be explained in two ways at least: the first reason would be a rupture of tradition whose cause has to be looked for outside the scientific environment of the epoch. This hypothesis is not improbable taking into account, first of all, the personality of Ibn al-Yāsamīn and his controversial behaviour which was strongly criticised by certain of his contemporaries, and taking into account also his close links with the Almohad power that was subsequently violently contested at the political, ideological and cultural levels [JULIEN 1969: 92-131; LAROUI 1970: 162-206]. The second reason, which is also probable and which might have reinforced the first one, is to be found in mathematical practice after Ibn al-Yāsamīn, that strongly bore the stamp of the mathematicians of Marrakech of whom we will speak later. One would be thus in the presence of the same phenomena of absorption that has already been observed earlier in the East, with the first Arab arithmetical writings of the 9th century, then in Moslem Spain with the writings of the 10th century. But, in the actual state of research, it is not possible to decide in favor of the one or the other hypothesis.

The contribution of Ibn Munᶜim

Aḥ;. mad Ibn Munᶜim was born in Dénia (on the east coast of Spain, near Valencia), but he spent a great deal of his life in Marrakech. In his epoch, he was considered one of the best specialists in Geometry and Number Theory. At the age of 30, he started to study Medicine which he practised in Marrakech in parallel to his activities of teaching and research. In Mathematics, Ibn Munᶜm would have published numerous works, dealing with such diverse subjects as Euclidean Geometry, Calculation, construction of magic squares, Number Theory and Combinatorics. [IBN ᶜABD AL-MALIK 1973: VI, 59-60]. But only one of his writings has come down to us. It concerns *Fiqh al-ḥ;. isāb* [The

science of Calculation] whose title does not reflect the diversity and the richness of content [DJEBBAR 1980, 1983a; LAMRABET 1981].

Before speaking of this content, one must notice the importance of the book for the history of mathematical activities in Moslem Spain. In fact, the author refers specifically to Andalusian scholars by citing their names, the titles of certain of their writings, and sometimes even passages from these writings. Thus it is due to him that we learn about the existence of the mathematician Ibn T;. āhir and that we discover another aspect of the activities of Ibn Sayyid, an important geometer of the 11th century. We have known for a few years, due to the testimony of his pupil, the philosopher Ibn Bājja (d. 1138), that this mathematician explored two very important areas of Geometry: that of the intersections of solids whose bases are curves different from conic sections, and that of curves obtained by the projection of these intersections of solids on a given plane [DJEBBAR 1984a: 84-91; ALAOUI 1983: 84-87]. Unfortunately, the geometrical works of Ibn Sayyid are lost and we do not know if they have circulated in the Maghreb. Ibn Munᶜim does not speak of these works since his book does not treat Geometry, but he specifically informs us about the contribution of Ibn Sayyid to the study of figurate numbers.

It is also due to Ibn Munᶜim that we may claim the presence, in the Maghreb from the 12th century onwards, of the book of al-Mu'taman (d. 1085), entitled *Kitāb al-istikmāl* [The book of perfection] which is essentially dedicated to Geometry but with a first chapter on Number theory [DJEBBAR 1984a ; HOGENDIJK 1986]. It seems indeed that this important work was transmitted from the Almohad capital. Later, Maimonides (d. 1204) taught the content of the book of al-Mu'taman in Cairo, and possibly also in Fez where he stayed for a time. During the 13th-14th centuries, certain chapters of the work continued to be studied in the extreme Maghreb, in particular by Ibn al-Bannā who refers to it explicitly in his *Risāthe fi t-taksi̇r* [Epistle on the calculation of areas] and, a little later, by Ibn Haydūr (d. 1413) [DJEBBAR 1986b].

With regard to the content of the *Fiqh al-h;. isāb*, its analysis shows that it does not always concerns a simple return to techniques and earlier mathematical results from the Andalusian tradition or transmitted by it. In fact in it one finds new trends and results whose origin is perhaps to be found in the activities of the Almohad capital or in the preoccupations of its intellectual environment. More precisely, one discovers in it, alongside the classical chapters on arithmetical operations, others like one on the study of figurate numbers, one on the determination of amicable numbers and, in particular, one on enumeration of all words of a language utilising a given alphabet. It is indeed here that Ibn Muniᶜd's most important contribution lies. He does not explain the reasons that motivated him to the study of this type of problem and which led him to dedicate a chapter of 19 pages that contains the important combinatorial propositions and trends which will be rediscovered, in Europe, only in the 16th and 17th century, in particular by Cardano (d. 1576),

Mersenne (d. 1648), Frénicle (d.1675) and Pascal (d.1662) [DJEBBAR 1983a: 25-44]. However one may reasonably suppose that it was the redynamisation of Arabic linguistic and grammatical activities in Marrakech that brought the problems of enumeration to the order of the day. On the other hand, it is reasonable to think that his way of solving these combinatorial problems by utilising the concrete model of threads of silk of different colours, finds its origin in the industrial or commercial environment of Marrakech.

We know that Ibn Munᶜim had students and that one of them, al-Qād;. ī ash-Sharīf (d. 1283-84) wrote a book, still not recovered, entitled *al-Qānūn fi l-h;.isāb* [The canon in Calculation]. But we do not know if this pupil incorporated the combinatorial results of his teacher in this book, or if he contented himself by teaching them to his own pupils and in particular to the most brilliant among them, the famous Ibn al-Bannā. However that may be, we find in the aforementioned work combinatorial preoccupations similar to those that led to the new results. In fact, in his little work, entitled *Tanbih al-albāb* [Advertisement to the intelligentsia], Ibn al-Bannā evokes explicitly one of the methods of Ibn Munᶜim, that of the arithmetic triangle, to enumerate all words which are possible to pronounce when one utilises the 28 letters of the Arab alphabet. But he does not stop here because he introduces three original contributions in this area: the most important concerns the announcement and the demonstration, for the first time to our knowledge, of the formula of factorials giving the combinations of n letters of a given alphabet taken p at a time, without utilising the arithmetic triangle, a result that will be established once again by Pascal three centuries later. In the second place, he establishes, as far as possible, the relations that exist between the figurate numbers of Nicomachus, the combinations of n objects taken p at a time and the sums of certain progressions of whole numbers. Finally, Ibn al-Bannā utilises the techniques or the trends of combinatorial type to solve certain problems outside mathematics and which lead to enumerations with constraints, as for example the determination of the number of possible readings of a given phrase, taking into account the rules of the Arab grammar, or of the number of prayers that a Moslem has to say to compensate for the forgetting of a certain number among them [DJEBBAR 1980: 67-98].

In the 14th century, we do not find new results in this domain among the commentators of Ibn al-Bannā, but simple references to the already established results, sometimes accompanied by comments. This is done by Ibn Haydūr in his book *at-Tamh;. is;. fi sharh;. at-Talkhis*;. [The in-depth commentary of the Abridgment] and in his handbook of calculation [GUERGOUR 1992]. One finds equally, and this is also important for the characterisation of the Magrebian mathematical tradition, the continuation of a certain combinatorial practice both inside and outside the mathematical field [DJEBBAR 1980: 99-112]. We do not yet have a detailed knowledge of this practice after the 14th century, but the little that we do know leads us to believe that we have here a domain that should still be the object of attentive study as it has not as yet

revealed all its secrets. Two precise examples allow us to assert this: the first concerns a rapid allusion to the enumeration of magical squares of a same type, of which one finds a trace in a manuscript of Euclid's *Elements* copied in the Maghreb. It is for the first time, to our knowledge that this problem is evoked in such an explicit way. Unfortunately, no reference to an earlier work on these enumerations is evoked by the copyist of the manuscript [Ms. Rabat, Hasaniya n° 1101: f.1a]. The second example concerns the commentary on the book of al-Qalas;. ādi *Kashf al-asrār ᶜan h;. urūf al-ghubār*. [The unveiling of the secrets relative to the dust ciphers] realised by T;. fayyash (d. 1914) a theologian of the central Maghreb who lived and taught in Beni Izguen. In the last chapter of his commentary, he discusses certain combinatorial aspects making implicit references to the work of Ibn al-Bannā.[Ms. Bibl. Béni Izguen: 445-50]. In our opinion, this evocation is not casual and could only be explained by the persistence of this combinatorial tradition that was born or established starting with the original contributions of Ibn Munᶜim and Ibn al-Bannā and that then was sustained in the Maghreb by manuals and teaching.

In concluding this chapter, we have to say that the relatively detailed presentation of the known writings of al-Qurashī, of al-H;. as;.s;. ār, of Ibn al-Yāsamīn and of Ibn Munᶜim does not mean that these writings were the only ones to have circulated and to have been studied in the 12th-13th centuries. The information we have at our disposal, shows the contrary. In fact, for the same period, one might evoke mathematicians sometimes as important as those of whom we spoke, like al-Qād;. ī ash-Sharīf, the pupil of Ibn Munᶜim à Marrakech, al-Qalᶜi (d. 1271) who lived equally in Bougie and who taught the Science of Inheritance [HAFNAWI 1982: 495-96], as also Ibn Ish;. āq at-Tūnusī (d. after 1218) known for his works in Astronomy [KING 1988]. Unfortunately, the mathematical writings of these scholars have not come to us and we cannot speculate on their contents. At most one may judge their importance from certain quotations of their writings or from certain testimonies on their stature as men of science.

MATHEMATICAL PRODUCTION IN THE MAGHREB DURING THE 14TH-15TH CENTURIES

In the history of scientific activities in the Maghreb, the 14th century constitutes a privileged moment both for the quantitative importance of the mathematical production that one may observe (in the light of the bibliographical testimonies and manuscripts which have survived), and equally for the content of this production and influence that it would have, during centuries, on the teaching of mathematics in the whole of North Africa and sometimes even in certain sub-Saharan regions.

According to the actual state of our knowledge concerning this period of the scientific history of the Maghreb, one may say that the majority of the mathematical production of this century is a return, in the form of

commentaries, summaries or developments, to a part of what had already been discovered or assimilated during the preceding centuries. New contributions are indeed exceptional, which may only confirm, in the case of mathematics, the conclusions arrived at by Ibn Khaldūn, in his *Muqaddima*, when he evoked the decline of certain scientific activities during his epoch.
This remark gives more importance to the mathematician that we will present now, as he appeared at the same time as one of the last innovators of the great Arab mathematical tradition and as one of the initiators of a new tradition of teaching of mathematics, based on the commentary. This tradition will involve whole the Maghreb and will extend itself even to Egypt.

The contribution of Ibn al-Bannā

Ibn al-Bannā was born in Marrakech in 1256. There he grew up and acquired an excellent training in various domains. But he also lived and taught for some time in Fez which became, after the fall of the Almohads, the capital of the dynasty of the Merinids, and which tried to rival, on an intellectual level, Marrakech, the only city which had the privilege of having been, for almost two centuries (1062-1248), the capital of the entire Maghreb, including vast sub-Saharan zones [LAROUI 1970: 147-185]. The information that we possess shows that a strong scientific tradition was established in this city, but the research is not sufficiently advanced to be able to describe the content of this tradition, its links with that of al-Andalus, and the profile of the men of science who lived there.
Returning to Ibn al-Bannā, we have to state immediately that we find ourselves in the presence of the last Maghrebian mathematician who was involved in research, to the extent that he tackled problems which were new for the epoch, and that he contributed with original solutions or advanced with new ideas. We have already mentioned his contribution to combinatorics which lies in the prolongation of research activities and preoccupations of Ibn Munᶜid. This contribution is not limited to what he has written in his *Tanbih al-albāb*. One finds equally interesting elements in his *Rafᶜ al-ḥijāb* [The lifting up of the veil] where he established the results that we mentioned earlier and where he lets us understand that in his epoch the problems of enumeration were not only related to the field of language. He also introduced a new trend in Algebra concerning the justification of the existence of solutions to the canonical equations of al-Khwārizmī and he would, according to the testimony of Ibn Haydūr, continue with a reflection on bases different from ten that would have been started by Ibn Munᶜim (in a work that has not come to us) [DJEBBAR 1980: 76-98; ABALLAGH 1988: 145-168, 517-543].
But, notwithstanding his exceptional qualities, noted by all biographers, the importance and prestige of Ibn al-Bannā did not only come from his mathematical works. In fact, our scholar distinguished himself from his Maghrebian predecessors by the richness and diversity of his production. Based on the inventory that was made, at the time, by Ibn Haydūr, we have

registered more than 100 titles of writings that are attributed to him, of which only 32 concern Mathematics and Astronomy [ABALLAGH & DJEBBAR 1995b], the others being dedicated to disciplines very distant from each other, like Linguistics, Rhetoric, Astrology, Grammatics and Logic.

Among his scientific writings, those related to the Science of Calculation seem to have assured the scientific notoriety of Ibn al-Bannā. His works on the Science of Calculation, that have come to us and been analysed are *Talkẖiṣ*. [The abridgment] , *Arbaᶜ maqālat fi l-ḥ*. *iṣāb* [The four works on Calculation] and the *Rafᶜ al-ḥ*. *ijāb* [The lifting up of the veil]. It is indeed the *Talkẖiṣ*. that illustrates the trend and conceptions of Ibn al-Bannā, at the level of the ordering of the chapters, the conciseness, the rigor, the formulation and the absence of all symbolism, although, with respect to this last point, there existed already a practice, in his epoch, both in the domain of the fractions, in that of the algebra of the polynomials and in that of equations.

The factors that led Ibn al-Bannā to inaugurate this type of very condensed manual, in opposition to the great collections of the 12th-13th centuries that we mentioned earlier, are perhaps not strictly pedagogical. This might have been the consequence of the whole training of Ibn al-Bannā and in particular of his mystical guiding that favoured a certain esoterism in addition to the difficulty and the abstract character of mathematical notions and techniques. However it may be, this character of the *Talkẖiṣ*. will lead numerous later mathematicians to write more or less detailed commentaries of this manual.

The encyclopedic character of the production of Ibn al-Bannā may have contributed to the social status of the mathematician who was honored by the Merinid power, which led him to leave Marrakech in order to instal himself for a time in Fez, at the invitation of the sultan of the epoch. This eminent position, from which he benefited in the Merinid capital, could only reinforce the authority that he had already acquired through his scientific works. One may also suppose that it was this double status, both scientific and social, that helped him to solve the problems that preoccupied his contemporaries, and which led him to publish an original book whose contents might be related, because of certain of its aspects, to Ethnomathematics. We are speaking about *Tanbīh al-albāb* that we have mentioned already on several occasions.

The first part of this little book contains the precise mathematical answers to questions that touch very varied domains of everyday life, like the composition of medicaments, the calculation of the drop of irrigation canals, the arithmetical explanation of a verse of the Koran concerning inheritance, the determination of the hour of the third daily prayer, the explanation of frauds linked to instruments of measurement, the enumeration of delayed prayers which have to be said in a precise order, the exact calculation of legal tax in the case of a delayed payment, etc. [ABALLAGH & DJEBBAR 1995b]. The second part, which belongs to the already ancient tradition of ludical and cultural mathematics, joins a collection of little arithmetical problems presented in the form of poetical riddles [DJEBBAR 1995b].

Before Ibn al-Bannā, mathematicians had also solved problems related to everyday life, but often the concrete aspect of the problem was no more than a dressing-up of arithmetical or abstract algebraic exercises. This does not happen in the case of the 17 problems exposed by Ibn al-Bannā. Having said this, and notwithstanding the originality of his book, Ibn al-Bannā was not the first to have solved problems specific to the Islamic city of the Middle Ages. To stay in the Maghreb, one may point out the case of a mathematician from Ifriqiya, al-Jṝt;. ālī (d. 1305) who wrote a work entitled *Kitāb maqāyṝs al-jurūh*;. [Book of the measures of wounds] in which he describes the method for measuring all types of wounds and the way to calculate the indemnities demanded by law as compensation to be paid to the wounded person [DJEBBAR 1995a]. We think that the two examples that we just mentioned are only one aspect of a rich tradition that deserves systematic research, both in mathematical writings and in the works of other disciplines like Law and Astrology.

The continuators of the tradition of Ibn al-Bannā

At the level of the great orientations of mathematical activity during the Middle Ages, Ibn al-Bannā appeared as the starting point of a whole tradition that extends to the different regions of North Africa and as far as Egypt, and which remained in Moslem Spain. This tradition is that of commentaries. There were thus more than fifteen relatively important works dedicated to the explanation or to the development, and sometimes even to the critique, of his little manual *at-Talkhṝs*;. . Certain among these commentaries were written by mathematicians of the extreme Maghreb: those of al-Mis;. rātī (14th c.), al-Muwāh;. idī (14th c.), Ibn Haydūr and of Ibn Ghāzī. One commentary was written by an Andalusian, Ibn Zakariyā', and some were written by either mathematicians of the Central Maghreb, like al-ᶜUqbānī (d. 1408), al-H;. abbāk (d. 1463) and Ibn Qunfudh, or by those from Ifriqiya, like al-Qalas;. ādī, or by Egyptians, like Ibn al-Majdī (d. 1447) and Ibn al-Hā'im (d. 1412) [ABALLAGH & DJEBBAR 1995b].

So far, none of these commentaries has been the object of a complete analysis, but a comparative study of their contents reveal both quantitative and qualitative differences. At the quantitative level, one finds short commentaries, like that of al-Mis;. rātī, a student of Ibn al-Bannā, who contents himself with explaining the definitions and the algorithms by means of examples without ever leaving the cadre of the *Talkhṝs*;. . At the other extreme, one finds true treaties for which the content of the *Talkhṝs*;. seems to be just a pretext or a guideline to permit their authors to expose in their own way, while sometimes criticizing Ibn al-Bannā severely, the themes dealt with by him and others he had deliberately abandoned. Among these treaties which significantly enrich the history of different mathematical disciplines, one may cite *Tamh;. ṝs;*.

[Profound study] of the Maghrebian Ibn Haydūr and *H;. āwi l-lubāb* [The collection of the marrow] of the Egyptian Ibn al-Majdi.

At the qualitative level, these commentaries are distinguishable one from the other by the utilisation or not of arithmetical and algebraic symbolism and by the recourse or not to the explanation or to the critique of certain definitions, to the demonstration of the propositions evoked by Ibn al-Bannā and to the justification of the validity of the algorithms that he exposed. One representative commentary of this category of works, whose authors did not confine themselves to the illustration of their objectives by means of examples, is that of al-ᶜUqbāni, a mathematician of the Central Maghreb who lived and taught in Tlemcen. It seems that this might be one of the last Maghrebian works containing demonstrations using the propositions of Euclid's *Elements* or instruments borrowed from works prior to the Arab mathematical tradition [HARBILI 1996: n° 7].

Having said this, the detailed analysis of the most important chapters of these commentaries permits us to make further remarks concerning both the nature of mathematics taught in the Maghreb in this epoch and its level:

In the first place, one notes that the level of the mathematics that is exposed is not lower than that of the previous period, but one does not recover certain themes that had been taught from the 10th century, such as the extraction of the approximate cubic root of a number, or the computation of new pairs of amicable numbers. This phenomenon was already perceptible in the work of Ibn al-Bannā and it was only to extend from the 14th century onwards.

In the second place, one does not notice any new contribution in these commentaries, either on the theoretical plane or at the level of the applications of earlier ideas and techniques. The most meaningful novelty is situated at the level of written expression with the progressive use of a relatively elaborate symbolism. The use of this symbolism which, as already mentioned, made its appearance in the writings of al-H;. as;.s;. ār and Ibn al-Yāsamin, was frozen or curbed during the entire 13th century and the first half of the 14th century. At least no mathematician of this epoch used it in the writings that came down to us. Indeed, even after this period, this symbolism does not seem to have been used by all commentators. In the case of the Extreme Maghreb, one finds it with al-Muwāh;. idi and Ibn Ghāzi, but not with al-Mis;. rāti and Ibn Haydūr. In the Central Maghreb, only Ibn Qunfudh and al-ᶜUqbāni used it [DJEBBAR 1980; GUERGOUR 1990; HARBILI 1996: n° 7]. In Ifriqiya, one finds this symbolism with al-Qat;. rawāni [LAMRABET 1981: 141-143; DJEBBAR 1986a: 118-120] and, later, with al-Qalas;. ādi [WOEPCKE 1858-59]. Outside the Maghreb, the same symbolism is utilised by the Egyptian Ibn al-Majdi, although it is absent in the works of his compatriot Ibn al-Hā'im [DJEBBAR 1980: 48]. Speaking about this, it would be interesting to see if this contradiction does not reflect two conceptions of teaching mathematics or, at least, of what came down of these two conceptions to the teachers of this epoch.

A final remark on the commentaries of the 14th-15th centuries concerns the wording and composition of their content and the style that is used. At this level, two trends are perceptible: the first is characterised by their stereotypical formulations which correspond to the usual style of mathematics, and the second, more rhetorical, is of a higher cultural level in the measure where the authors prolong and complete their mathematical explanations with grammatical, literary or philosophical commentaries. This trend contributes nothing on the mathematical level, but it informs us about some aspects of the culture of the epoch and the progressive intervention of this culture in certain technical domains.

In the domain of Algebra, different sources inform us that the works of Ibn Badr [SANCHEZ-PEREZ 1916], al-Qurashī, Ibn al-Yāsamīn and Ibn al-Bannā continued to be taught in cities of the Maghreb such as Fez, Tlemcen, Sebta and Tunis. However only the poem of Ibn al-Yāsamīn seems to have been in favour with the commentators, both in the Maghreb and in Egypt. Among the commentaries that circulated from the 14th century onwards, one may cite those of Ibn Qunfudh and al-Qalasādī in the Maghreb [GUERGOUR 1990; ZEMOULI 1993] and those of al-Māradinī (d. 1501) and Ibn al-Hā'im (d. 1412), in Egypt [SHAWQI 1987; SOUISSI 1983b].

Having said this, and if one excludes only one commentary of *Rafᶜ al-ḥ;. ijāb* realised by Ibn Haydūr, no other work on Calculation or on Algebra in the 12th-14th centuries motivated the Maghrebian commentators. Indeed until today, we have not found any mention of an eventual commentary of the great works on Calculation of al-H;. as;.s;. ār and of Ibn al-Yāsamīn or of the book of Ibn Munᶜim. Similarly we have still not discovered any Maghrebian commentary on the classics of Arab Algebra of the 9th-10th centuries, which nevertheless circulated in Andalusia and in the Maghreb.

The explanation of this phenomenon is not simple: one may look for the reasons either in a lowering of the general standard of teaching, or in the stopping of research activity, or in the lack of interest in the theoretical aspects of scientific disciplines. These causes are in fact linked one to another, and all can be traced to factors beyond scientific activity itself which the great Maghrebian historian of the 14th century, Ibn Khaldūn, had already, in his epoch, understood the effects on society as a whole and, from there, on the dynamism of scientific and cultural activities of the entire Moslem West.

Whatever the cause, one notes that difficult Maghrebian works, or those having that reputation, are left by the commentators or are only used to better clarify the explanation of this or that passage of the *Talkhīs;.* of Ibn al-Bannā.

One notes also that there is, from this epoch onwards, a change in the scientific production of the Moslem West, even at the level of references to the works. One continues certainly to evoke Euclid, Nicomachus and al-Khwārizmī, but it is foremost the authors of the Maghreb or of Moslem Spain who are cited in the commentaries of the 14th-15th centuries that have come down to us.

THE MATHEMATICIANS OF THE MAGHREB AFTER THE 15TH CENTURY

To our knowledge, mathematical production of the period that extends from the beginning of the 16th century to the end of the 19th has not as yet been the subject of any global study. The remarks we are going to present briefly are based essentially on bio-bibliographical elements and on the analysis of some texts lithographed during the 19th century or recently edited [KHATTABI 1987].

The known number of mathematicians (or teachers of mathematics), who lived in the Maghreb after the 15th century surpasses 150. The disciplines that they taught, or that they dealt with in the works that came down to us, are metrical Geometry, Calculation, magical squares and the distribution of inheritance, as far as Mathematics is concerned, and the calculation of time, the determination of the direction of Mecca and the description of astronomical instruments, as far as Astronomy is concerned [SUTER 1900; BROCKELMANN 1937-49; LAMRABET 1994].

Restricting ourselves to Mathematics, and in the light of the texts that we have been able to study, we may affirm that the content of this production differs from the earlier mathematical writings in form and standard. One finds poems, like those of al-Akhḍarī (d. 1576), al-Wansharīsī (d. 1548) and ar-Rasmūkī (d. 1721), glosses or commentaries like those of Ibn al-Qāḍī (d. 1616) and Muḥammad Bannīs (d. 1798), and summaries like those of al-Fāsī (d. 1685). Concerning the level of these writings, it is lower than that of the works of the 15th century, which are themselves much less rich, with respect to ideas and techniques, than the works of the 13th-14th centuries. This corresponds well to what one may observe in others sectors of intellectual activity in the whole of North Africa.

Among the internal factors that might be the origin of this long process of decline of mathematical activities, there is the slowing-down and later the termination of research, and the allure of a qualitative change in the content of the works of teaching that were progressively limited to the exhibition of techniques and results without any demonstration. These internal factors were themselves induced by external factors which belong either to the economical sphere, such as the drying out of the African gold route or the loss of control of sea routes, or the political sphere, such as the repeated offensives against the coast of North Africa by the new powers of southern Europe, i.e. Spain and Portugal and certain Italian city-states [LAROUI 1970 : 211-223].

It is thus not surprising that this long process of degradation had indirect effects on mathematics through the progressive contraction of their domain of activity and of their field of application. Thus, the only activities which remained for the mathematicians, different from teaching and elaboration of manuals, were those of mathematical practices directly related to activities or preoccupations of a religious character, such as the distribution of inheritance and donations to rightful claimants, the determination of time for the fixing of

moments for prayer, or the construction and use of astronomical instruments, like the quart of sine and the astrolabe.
As a consequence of the stopping of research in Mathematics, the contraction of its field of application and the reduction of the content of the programmes, one sees the appearance and development, both in the Maghreb and in Egypt, of the practice that consists of publishing, on a single mathematical subject, a series of works that only differ in style (poetry or prose), volume (book or summary) or form (detailed commentary or glosses). This tendency was already very obvious from the second half of the 15th century in the case of al-Qalas;. ādī in Ifriqiya, and Ibn Ghāzī in the Extreme Maghreb. It continued at least until the 18th century, as the production of ar-Rasmūkī (d. 1775) shows. But, notwithstanding the absence of originality of these numerous writings, they constitute precious material for the history of scientific teaching in North Africa.

CONCLUSION

At the end of this review of ten centuries of mathematical activities in the Maghreb, it seems us useful to make some remarks of a general nature concerning the content of these activities and what has been transmitted to other regions of Africa.
The first remark concerns certain non-Arab computational survivals. The information presented shows that mathematical practice in the Maghreb inscribes itself essentially in the Arab tradition. But, this does not mean that the practice was unique. One notes in fact, in the area of computation, and that seems to be a peculiarity of the Extreme Maghreb, the survival, from the pre-Islamic epoch, of a computational practice that uses symbols called the *ciphers of Fez*. These symbols distinguish themselves from the *ghubār ciphers*, i.e. today's ciphers, by their number and form. The persistence of this practice was such that mathematicians like Ibn al-Bannā in the 14th century, and others less prestigious, wrote manuals to explain its principles and use [DJEBBAR 1987: 239-40].
The second remark concerns the place of the Maghreb in the mathematical tradition of the Moslem West. It is clear, in the light of the information that we have presented, that it was essentially the cities of the Extreme Maghreb, in particular Sebta, Fez and Marrakech, which took the relay from Moslem Spain in mathematical activity from the 12th century onwards until the end of the 14th. After this period there was a greater intervention of two other scientific poles: Tlemcen in the Central Maghreb and Tunis in Ifriqiya. The reasons why this relay did not take place before the 12th century are not easy to determine with certainty. Among the reasons is probably the proximity of Moslem Spain and its dynamism during the 9th-11th centuries, which regularly attracted the elite who were first trained in the cities of the Maghreb and who then installed themselves provisionally or definitively in a city of al-Andalus. As examples, one may cite the case of al-Wahrānī (ca. 1037) for the Central Maghreb

[SUTER 1900: n° 251], al-Kalāᶜi (d. 1111) for Ifriqiya [DJEBBAR 1988b: 64] and Ibn Yāsīn (9th c.) for the Extreme Maghreb [SUTER 1900: n° 106]. This phenomenon was able to influence the constitution of scientific foyers of high level negatively in the Maghrebian metropoles.

Other probable causes are to be found in the economic sphere where the Maghreb appeared, until the Almoravid epoch, much more as a relay than a pole attracting wealth and know-how. From the 12th century onwards, it seems likely that diverse factors were united to allow the Maghreb to become a pole. But, it is also from this epoch onwards that Arab scientific activity taken as a whole started to show signs of running out of breath and slowing-down in the metropolis of other regions. Having said this, the 12th-13th centuries have not as yet revealed all their secrets and it is very likely that this short period during which the Maghreb realised its political unity has been, in the field of mathematics, still more fruitful than is apparent from the few works that came down to us and that we havepresented briefly.

The third and last remark concerns the role of the Maghreb in the diffusion of the Arab mathematics. This diffusion is first effected, it seems, towards southern Europe in a direct manner as a consequence of translations, such as the presently unique example of the Hebrew translation of the *Kitāb al-bayān* of al-H;.;. as;.s;. ār, or in an indirect manner by the assimilation of local teaching realised in Arabic followed by the elaboration of manuals or treaties in Latin or Hebrew. The most famous example that illustrates this phenomenon and is still little studied, is that of Fibonacci. As he says himself, this Italian scholar was trained, when very young, in Bougie, one of the Maghrebian scientific poles of the 12th century and, later, he reproduced, in his *Liber Abbaci*, certain aspects of the Magrebian mathematical tradition and in particular with respect to the symbolism and computation of fractions [VOGEL 1970-80: 605-609].

However according to our information, the transmission of mathematical writings of the Maghreb essentially went in two other directions. The first is that of the East, more precisely Egypt, where the writings of al-H;. as;.s;. ār, Ibn al-Yāsamīn and Ibn al-Bannā circulated or were the object of commentaries. This fact is confirmed by the testimony of the encyclopedist of the 14th century, Ibn al-Akfānī [IBN AL-AKFĀNĪ 1990], and by the commentaries of the *Talkhīs*;. . The content and the modalities of this transmission have still not been the object of an in-depth study , but a comparative analysis of the documents that exist might clarify more about this little known phenomenon.

The third direction in which the transmission went is that of Sub-Saharan Africa. This phenomenon is, unfortunately, still less known than the two preceding ones. It seems that it was the Extreme Maghreb that was the principal relay for this transmission that essentially concerned the Science of Computation and Astronomy. We have a first confirmation of this through the manuscripts that are today in the Ah;. mad Bābā library in Timbuktu. Among

these manuscripts, only one, treating computation, is attributed to a scholar from the region. It concerns Ah;. mad Bābir al-Arawānī, a mathematician native from Arawān (Mali) who lived after the 16th century, as he refers in his writing to an arithmetical poem, *ad-Durra al-bayd;. ā'* [The white pearl] of a mathematician of the Central Maghreb called al-Akhd;. arī (d. 1575) [Ms. Bibl. Ahmad Baba n° 3027]. The other manuscripts are either mathematical poems, like those of as-Samlālī and ar-Rasmūkī, both the Extreme Maghreb [LAPOUSTERLE 1990], or also astronomical writings, like the *Kitāb tarh;. il ash-shams* [The book of the movement of the sun] of Ibn al-Bannā.

We cannot appreciate the importance of this diffusion on the basis of the rare elements that are in our possession, and a number of questions concerning scientific activity South of the Sahara remain to be answered and still will for some time. However, if one takes into account the information that came down to us in respect of the cultural history of this region [BATILY 1989; DRAMANI-ISSIFOU 1982; NIANE 1975; EL-FASI & HRBEK 1990; NIANE 1975a, 1975b, 1985], it seems reasonable to think that the circulation of scientific writings was more important than the documents accessible today suggest. Likewise one may suppose that students native of Sub-Saharan zones, perhaps had the possibility to move themselves to the North, for example to undertake a pilgrimage to Mecca or for other reasons, as happened with Ah;. mad Bābā at-Tambuktī (d. 1627). One or other of these circumstances would have given these students the opportunity to perfect themselves under the guidance of known professors, before themselves becoming teachers or authors of works, as shown by the example of al-Arawānī that we mentioned earlier, and that of al-Katsināwī (d. 1741), a scholar native of Katsina (Nigeria) who lived for a time in Cairo and who specialised in the construction of magic squares [ZASLAVSKY 1973: 138-151; KANI 1992b: 17-36; GERDES 1992: 17; SESIANO 1994].

However, we cannot evaluate objectively either the content of the scientific production or the nature and intensity of the exchanges between the men of science of the different regions of North Africa, until new investigations of as yet inaccessible documents concerning all aspects of the intellectual life of the cities of these regions. Having said this, and waiting for the discovery and comparative analysis of these documents, we hope that the few elements we have presented already allow us to convince the reader of the importance of the scientific heritage of this part of Africa.

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