

1. OBJECTIVES

The A.M.U. Commission on the History of Mathematics in Africa (AMUCHMA), formed in 1986, has the following objectives:

- a. to improve communication among those interested in the history of mathematics in Africa;
- b. to promote active cooperation between historians, mathematicians, archaeologists, ethnographers, sociologists, etc., doing research in or related to the history of mathematics in Africa;
- c. to promote research in the history of mathematics in Africa and the publication of its results in order to contribute to the demystification of the still dominant eurocentristic bias in the historiography of mathematics;
- d. to cooperate with any and all organisations pursuing similar objectives.

The main forms of activity of AMUCHMA are as follows:

- a. publication of a newsletter;
- b. setting up a documentation centre;
- c. organisation of lectures on the history of mathematics at national, regional, continental and international congresses and conferences.

2. ON THE HISTORY OF MATHEMATICS IN AFRICA SOUTH OF THE SAHARA*

- * slightly adapted version of a paper presented by **Paulus Gerdes** at the **Third Pan-African Congress of Mathematicians**, Nairobi, 20-28 August 1991. This paper forms part of the study "**Recent research on the History of Mathematics in Africa: an overview**" by A.Djebbar and P.Gerdes

Introduction

In her classical study "**Africa Counts: number and pattern in African culture**" (1973a; review by Wilder, 1976), C.Zaslavsky presented an overview of the available literature on the history of mathematics in Africa south of the Sahara. She discussed written, spoken and gesture counting, number mysticism, concepts of time, numbers and money, weights and measures, record-keeping (sticks and

strings), mathematical games, magic squares, graphs, and geometric form, and Crowe contributed a chapter on geometric symmetries in African art. Since the publication of Zaslavsky's overview many scholars, students, teachers and laymen alike - both in Africa and abroad - have become interested in the mathematical heritage of Africa south of the Sahara. The African Mathematical Union (**AMU**) included a History section in the 2nd Pan-African Congress of Mathematicians (Jos, Nigeria, 1986) with as one of its purposes "to encourage more reports and exchanges of references and ideas on historical studies of African mathematics" (Shirley, 1986b, 3). The success of this section stimulated the formation of the AMU Commission on the History of Mathematics in Africa (**AMUCHMA**). In order to stimulate research on the history of mathematics in Africa in general, and to promote the divulgation of the research findings and the exchange of information in this field, AMUCHMA has published since 1987 a newsletter in English, French and Arabic. In this paper an overview of research (findings) on or related to the history of mathematics in Africa south of the Sahara is presented. Topics like counting and numeration systems, numerology, mathematical games and puzzles, geometry, graphs, Islam and mathematical development, international connections, and history of mathematics curricula will be included. Attention will also be paid to the objectives of research in the history of mathematics in Africa, to methodology, to the relationship with ethnomathematical research and to the uses of research findings in mathematics education. Some possible directions for further research will be identified.

Why study the history of mathematics in Africa south of the Sahara?

There are many reasons which make the general study of the history of mathematics both necessary and attractive (see e.g. Struik, 1980). There exist important additional reasons which make the research on the history of mathematics in **Africa south of the Sahara** indispensable. African countries face the problem of low 'levels' of attainment in mathematics education. Math anxiety is widespread. Many children (and teachers too?) experience mathematics as a rather strange and useless subject, imported from outside Africa. One of the causes thereof is that the goals, contents and methods of mathematics education are not or not sufficiently adapted to the cultures and needs of the African peoples, as stresses the first Secretary-General of the AMU Commission for Mathematical Instruction (Eshiwani, 1979, 346; cf. Eshiwani, 1983; Jacobsen, 1984). Today's existing African educational system is "unadapted and elitist" and "favours foreign consumption without generating a culture that is both compatible with the original civilization and truly promising" (Ki-Zerbo, 1990, 4; cf. El-Tom on mathematics education and the selection of élites, 1984, 3). The delegates to the Vth Conference of Ministers of Education and those Responsible for Economic Planning in African Member States declared that educational policy should be designed to "restore to their rightful status the African

cultural heritage and the traditional social and human values that hold potential for the future " (MINEDAF, 1982, 41). The mathematical heritage of the peoples of Africa has to be valued and African mathematical traditions should be 'embedded' into the curriculum (Cf. e.g. Ale, 1989; Doumbia, 1984, 1989b, Gerdes, 1985a, 1986a, 1986b, 1988d, 1990c; Langdon, 1989, 1990; Mmari, 1978; Njock, 1985; Shirley, 1986a, 1986b). And as this scientific legacy of Africa south of the Sahara is little known, research in this area constitutes a **challenge** to which an urgent response is necessary (Njock, 1985, 4). Also African-Americans and minorities of African descent all over the world feel the need to know their cultural-mathematical heritage (Campbell, 1977; Frankenstein & Powell, 1989; Zaslavsky, 1973, etc.; Ratteray, 1991). More generally, both in highly industrialised and in Third World countries it is becoming more and more recognised that it is necessary to **multi-culturalise** the mathematics curriculum in order to improve its quality, to augment the cultural confidence of all pupils and to combat racial and cultural prejudice (cf. e.g. D'Ambrosio, 1985a; Ascher, 1984; Bishop, 1988a, b; Joseph, 1987; Mellin-Olsen, 1986; NCTM, 1984; Nebres, 1983; Zaslavsky, 1989a, 1991).

Broad conception of 'history' and 'mathematics'

Most histories of mathematics devote only a few pages to Ancient Egypt and to northern Africa during the 'Middle Ages'. Generally they ignore the history of mathematics in Africa south of the Sahara and give the impression that this history either did not exist or, at least, is not knowable / traceable, or, stronger still, that there was no mathematics at all south of the Sahara (cf. Lumpkin, 1983; Njock, 1985). "Even the Africanity of Egyptian mathematics is often denied" (Shirley, 1986b, 2). Prejudice and narrow conceptions of both 'history' (cf. Ki-Zerbo, 1980, General Introduction) and of 'mathematics' form the basis of such (eurocentric) views (cf. Joseph, 1987, 1991).

At the 17th International Congress of Historical Sciences, Humphrey (1990, 4) stressed that "Any narrow definition of science in modern terms would make it difficult for us to understand its origins and the variable forms it has taken in different cultures". In the case of mathematics, authors like Ale, D'Ambrosio, Ascher & Ascher, Bishop, Doumbia, Gerdes, Njock, Shirley and Zaslavsky consider 'mathematics' as a pan-cultural phenomenon and propose a broad conception, including counting, locating, measuring, designing, playing, explaining, classifying, sorting...

Pioneer study

Zaslavsky's '**Africa Counts**' is a pioneer work in the area of the history of mathematics south of the Sahara. She offers her book as "a preliminary survey of a vast field awaiting investigation" (1973a, vi). Her task was not an easy one: in face of "the inadequacy of easily accessible material...", she had to search "the literature of many disciplines - history,

economics, ethnology, anthropology, archaeology, linguistics, art and oral tradition - ..." (1973a, vi).

She used a broad perspective on mathematics; her study deals with, what she calls, the '**sociomathematics**' of Africa: she considers "the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of mathematics" (1973a, 7). The concept of sociomathematics may be considered a forerunner of the concept of **ethnomathematics**. It is ethnomathematics as a discipline that studies mathematics (and mathematical education) as embedded in their cultural context - the (development of) different forms of mathematical thinking which are proper to cultural groups, like ethnic, professional, and age groups. For the (possible) relationships between ethnomathematics and the history of mathematics, see (in general) D'Ambrosio (1985b) and (in the case of Africa) Shirley (1986b) and Gerdes (1990e).

The application of historical and ethnomathematical research methods has contributed, as will be shown, to the knowledge and understanding of the history of mathematics in Africa, or, at least, of some further mathematical elements in African traditions, in addition to the information gathered in '**Africa Counts**'.

The beginnings

Zaslavsky presented as early evidence for (proto-)mathematical activity in Africa a bone dated at 9000-6500 B.C., dug up at Ishango (Zaire). The bone has what appear to be tallying marks on it, notches carved in groups. The bone's discoverer, De Heinzelin, interpreted the patterns of notches as an "arithmetical game of some sort, devised by a people who had a number system based on 10 as well as a knowledge of duplication and of prime numbers". Marshack, on the contrary, explains the bone as early lunar phase count. Their views, summarized in (Zaslavsky, 1973a, 17-19), are reproduced recently in (Fauvel & Gray, 1987, 5-7). Later, the dating of the Ishango bone has been reevaluated, from about 8000 B.C. to 20,000 B.C. (Marshack, 1991). Zaslavsky (1991b) raises the question "who but a woman keeping track of her cycles would need a lunar calendar?" and concludes that "women were undoubtedly the first mathematicians!".

Bogoshi, Naidoo & Webb report in 1987 on a still much older "mathematical artefact": "A small piece of the fibula of a baboon, marked with 29 clearly defined notches, may rank as the oldest mathematical artefact known. Discovered in the early seventies during an excavation of Border Cave in the Lebombo Mountains between South Africa and Swaziland, the bone has been dated to approximately 35000 B.C.". They note that the bone "resembles calendar sticks still in use today by Bushmen clans in Namibia" (1987, 294).

A research project looking for numerical representations in San (Bushmen) rock art has recently been started by Martinson (University of the Witwatersrand, South Africa). From the surviving San hunters in Botswana - "the oldest pattern of life found in the world today..." - , Lea

and her students at the University of Botswana have collected information. Her papers describe counting, measurement, time reckoning, classification, tracking and some mathematical ideas in San technology and craft. The San developed very good visual discrimination and visual memory as needed for survival in the harsh environment of the Kalahari desert (Lea, 1987, 1989, 1990a, 1990b).

Numeration systems

Zaslavsky's discussion of written, spoken and gesture counting and numeration systems is primarily based on Almeida (1947: Guine Bissau), Armstrong (1962: Yoruba, Nigeria), Atkins (1961), Delafosse (1928), Herskovits (1939: Kru, Liberia -Ivory Coast), Lagercrantz (1968: tally-systems), Mann (1887: Yoruba), Mathews (Northern Nigeria), A.V.O. (1964: Hima, Uganda), Raum (1938), Schmidl (1915), Thomas (1920), Torrey (1963), Williamson (1943: Dabida, Kenya). In the meantime other sources came also to the fore, like Seidenberg (1959, 1963, 1976), Santos (1960: Tchokwe, Angola), Hazoume (1983: Gun, Gen and Bariba). During the last years, a whole series of research projects on spoken and written numeration systems in Africa is being carried out, e.g. on:

- * counting in traditional Ibibio and Efik societies (I.O.Enukoha, University of Calabar, Calabar, Nigeria);
- * numeration among the Fulbe (Fulani) (S.O.Ale, Ahmadu-Bello-University, Bauchi, Nigeria);
- * pre-Islamic ways of counting (Y.Bello, Bayero University, Nigeria);
- * counting in Nigerian languages (Ahmadu-Bello-University, Zaria; cf. Shirley, 1988b);
- * pre-colonial numeration systems in Burundi (J.Navez, University of Burundi, Bujumbura);
- * learning of counting in Côte d'Ivoire (cf. Zepp, 1983c);
- * numeration systems used by the principal linguistic groups in Guinea (S.Oulare, University of Conakry);
- * counting among the various ethnic groups in Kenya (J.Mutio, Kenyatta University, Nairobi);
- * traditional counting in Botswana (H.Lea, University of Botswana, Gaborone);
- * numeration systems and popular counting practices in Mozambique (Higher Pedagogical Institute, Maputo / Beira; cf. Soares, 1991).

A important study - from the point of view of its contents and the methodological debate it initiates - is E.Kane's doctoral dissertation (1987) on **"The spoken numeration systems of west-atlantic groups and of the Mandé"**. Kane (Cheik-Anta-Diop-University, Dakar-Fann, Senegal) analyses numeration in about twenty languages spoken in Senegal. He realised the necessity of basing his research on ethnomathematics, trying to understand mathematical ideas in relationship to the general culture in which they are embedded.

Therefore he did preparatory research in four domains: African linguistics, history of numeration systems, works of Africanists and African languages spoken in Senegal (as understood by interviewing many speakers of the same and different languages). He shows that spoken numeration systems, like the one of the Mandé, are susceptible to reform and evolution. Kane develops a methodology for the analysis of numeration systems that is adapted to the specificities of 'oral cultures'.

Number symbolism

Zaslavsky dedicated a chapter to number symbolism, superstitions and taboos on counting (1973, 52-57; cf. Williamson & Timitimi (1970, Ijo, Nigeria)). Vergani (1981) wrote a Ph.D.thesis on number symbolism among the Tchokwe of Angola (see below: networks). Ojode (1988) published a paper on the number 3 in African lore, highlighting the sacredness, mysticism and taboos attached to it (Cf. also Nicolas, 1968). In Page (1987) objects of African art, mostly from the Yoruba (Nigeria) are analysed in function of the involved repetitions. The twofold objects evoke the most usual dichotomies: good/bad, life/death; the threefold objects evoke sometimes a hierarchy; the fourfold objects may be associated with the directions in space. Probably by searching systematically the ethnographical literature as well as romances, biographies, etc. a lot more information on number symbolism in African cultures may be found. For instance, the anthropological study of Thornton explains the significance of the number 9 among the Iraqw of Tanzania (1980, p.96, 167, 183). Number symbolism may have a rational basis. E.g. Makua basket makers in northern Mozambique call odd numbers or odd quantities of plant strips 'ugly', and they have good reasons to do so (Ismael, 1991; cf. earlier discussion of 'even' and 'odd' numbers in basketry in Gerdes, 1985a). Certainly, further collecting of oral data may throw new light on African numerology.

Riddles and puzzles

Zaslavsky (1973, 109-110) presents a riddle from the Kpelle (Liberia) about a man who has a leopard, a goat, and a pile of cassava leaves to be transported across a river, whereby certain conditions have to be satisfied: The boat can carry no more than one at a time, besides the man himself; the goat cannot be left alone with the leopard, and the goat will eat the cassava leaves if it is not guarded. How can he take them across the river? Ascher (1990) places this river-crossing problem in a cross-cultural perspective and analyses mathematical-logical aspects of story puzzles of this type from Algeria, Cape Verde Islands, Ethiopia, Liberia, Tanzania and Zambia. More difficult to solve is an 'arithmetical puzzle' from the Valuchazi (eastern Angola and northwestern Zambia), recorded and analysed by Kubik (1990): "This..dilemma tale is about three women and three men who want to cross a river in order to attend a dance on the other side. With the river between them there is a boat with the capacity for taking only two people at one time. However, each of the

men wishes to marry all the three women himself alone. Regarding the crossing, they would like to cross in pairs, each man with his female partner, but failing that any of the other men could claim all the women for himself. How are they crossing?" (Kubik, 1990, 62). In order to solve the problem or to explain the solution, auxiliary drawings are made in the sand. Fataki (1991) describes riddles he learnt as a child in Uganda.

Art and symmetries (see also below: cross-cultural psychology)

Njock (University of Yaoundé, Cameroon) characterises the relationship between African art and mathematics as follows: "Pure mathematics is the art of creating and imagining. In this sense black art is mathematics" (1985, 8).

Mathematicians have analysed mostly symmetries in African art. Symmetries of repeated patterns may be classified on the basis of the 24 different possible types of patterns which can be used to cover a plane surface (cf. the so-called 24 plane groups due to Federov, 1891). Of these, seven admit translations in only one direction and are called strip patterns. The remaining 17 that admit two independent translations are called plane patterns. In chapter 14 of (Zaslavsky, 1973), Crowe applies this classification to decorative patterns that appear on the raffia pile cloths of the Bakuba (Zaire) (Cf. Crowe, 1971), on Benin bronzes, and on Yoruba adire cloths (Nigeria), showing that all seven strip patterns occur and many of the plane patterns. Crowe continued this research and published a catalog of Benin patterns (Crowe, 1975) and a symmetry analysis of smoking pipes of Begho (Ghana) (Crowe, 1982a; cf. also Crowe, 1982b). In Washburn and Crowe (1988) a number of patterns from African contexts are classified in the same way. Recently Washburn (1990, ch.5) showed how a symmetry analysis of the raffia patterns can differentiate patterns produced by the different Bakuba groups. Although the use of the crystallographic groups in the analysis of symmetries in African art attests and underlines the creative imagination of the artists and artisans involved and their capacity for abstraction (cf. Meurant, 1987), these studies do not focus on how the artists and artisans themselves classify and analyse their symmetries. This is a field open for further research. Zaslavsky (1979) gives some examples of strip and plane patterns, and of bilateral and rotational symmetries, occurring in African art, architecture and design. Why do symmetries appear in human culture in general, and in African craftwork and art, in particular? This question is addressed by Gerdes in a series of studies. He analyses the origin of axial, double axial, and rotational symmetry of order 4 in African basketry (Gerdes, 1985a, 1987, 1989a, 1990c, 1991c). In (Gerdes, 1991b) it is shown how fivefold symmetry emerged quite "naturally" when artisans were solving some problems in (basket)weaving. The examples chosen from Mozambican cultures range from the weaving of handbags, hats, and baskets to the fabrication of brooms.

Langdon (1989, 1990) describes the symmetries of 'adinkra' cloths (Ghana) and explores possibilities for using them in the classroom. In a similar perspective, Harris (1988) describes and explores not only the printing designs on plain woven cloths from Ghana, but also symmetries on baskets from Botswana and 'buba' blouses from the Yoruba (Nigeria).

Games

Among the games with mathematical 'ingredients' referred to in

(Zaslavsky, 1973, 102-136) are counting rhymes and rhythms, three-in-a-row-games, arrangements, games of chance and board games. Zaslavsky (1982) gives more information on three-in-a-row-games in Africa. Russ (1984; review by Crowe, 1987) presents the rules and a brief history of 'mancala' games (cf. Townshend, 1979), also known as Ayo, Bao, Wari, Mweso, Ntchuva, etc. Zaslavsky (1989, AMUCHMA3, p.6) suggests that it could be important in the reconstruction of the history of mathematical thinking in Africa to investigate further mathematical aspects of traditional games. As a starting point, she indicates, along with Russ (1984), the following publications: Béart (1955), Centner (1963), Driedger (1972), Klepzig (1972), and Pankhurst (1971). Béart's **"Games of West Africa"** has been reviewed by Doumbia (1989a). To this list, Nsimbi's **"Omweso: a game people play in Uganda"** (1968), Mizoni's **"Strategic games in Cameroon and their mathematical aspects"** (1971), Deledicq & Popova's **"Wari and solo, the African calculation game"** (1977), Ballou's **"Rules and strategies of the awélé game"** (1978) and Crane's **"African games of strategy"** (1982) may be added. Crane informs about some of the most common types of African games involving strategy and mathematical principles, like games of alignment (Shisima (Kenya), Achi (Ghana), Murabaraba (Lesotho)), 'struggle-for-territory' games (Sega (Egypt), Kei (Sierra Leone), and 'Mancala' games, both two-row versions (Oware (Ghana), Awélé (Ivory Coast), Ayo and Okwe (Nigeria)) and four-row versions (Omweso (Uganda), Tshisolo (Zaire)). Bell and Cornelius (1988) give some information on Achi (Ghana), Dara (Nigeria), Sega (Egypt) and on 'Mancala' games. Retschitzki (1988) and N'Guessan (1988) analyse the learning of strategies and tactics of the 'awélé' game. Important is the research of Doumbia (1989b) and her colleagues at the Mathematical Research Institute of Abidjan (Ivory Coast). Their work on traditional African games deals with classification, solution of mathematical problems posed by the games and analysis of the possibilities of using these games in the mathematics classroom. Their conclusion - as revealing as it is - that rules of some games, like Nigbé Alladian, show a traditional, at least empirical knowledge of probabilities, will certainly stimulate further research. Vergani (Open University, Lisbon) prepares a monograph on mathematical aspects of intellectual games in Angola. Mve Ondo (Omar-Bongo-University, Gabon) published recently (1990) a study on two 'calculation games', i.e. the 'Mancala' games, Owani (Congo) and Songa (Cameroon, Gabon, Equatorial Guinea). The possible relationship between visual memory and concentration as necessary for success in many African games (cf. Paul, 1971) and the development of mathematical ideas also deserves further attention.

Geometry and architecture

Chapter 13 of (Zaslavsky, 1973a, 155-171) (Cf. Zaslavsky, 1989) is dedicated to geometric form in architecture. More information on the geometric shapes and on the ornamentation of traditional African buildings may be found in (Denyer, 1978). (Anon., 1987) presents a

bibliography on African architecture. Prussin calls attention to the fact that in West Africa the mathematician-scholar and the architectural design-builder might often be the same person (1986, 208). She refers to the relationship between magic squares and the structure of domes and remarks that "a number of 'adinkra' [Ashanti, Ghana] stamp patterns directly associated with Islam were also used in the architectural setting" (Prussin, 1986, 240). Rohrman (1974) and Matthews (1974) describe house decoration and mural painting in southern Africa. A publication of NTTC (1976) gives a catalog of geometric patterns used on house walls in Lesotho. These studies may serve as a starting point for further research on geometry and ornamentation of buildings. Eglash and Broadwell (1989) are interested in possible relationships between modern fractal geometry and traditional (knowledge about) settlement patterns in Africa. Gerdes (1985a) describes the geometrical know-how used in laying out the circular or rectangular house plans in Mozambique. His student Mahanjane (1989) uncovers geometrical knowledge applied in the construction of traditional granaries for maize and beans in the Gaza province in southern Mozambique.

Uncovering 'hidden' mathematical ideas: geometrical form

Many 'mathematical' ideas and activities in African cultures are not explicitly mathematical. They are often intertwined with art, craft, riddles, games, graphic systems, and other traditions. The mathematics is often 'hidden'. How may this 'hidden' knowledge be uncovered? And as some traditions are nowadays (becoming) obsolete, this 'uncovering' often also means a tentative reconstruction of knowledge as it existed in the past. Gerdes (1985a) explores the concept of 'hidden' mathematics and develops some methods in order to 'uncover' and reconstruct 'hidden' geometrical thinking (cf. also Gerdes, 1986a, 1986b, 1987, 1990c, 1990e). One of these methods may be characterised as follows: when analysing the geometrical forms of traditional objects - like baskets, mats, pots, houses, fishtraps - the researcher poses the question: why do these material products possess the form they have? In order to answer this question, the researcher learns the usual production techniques and tries, at each stage of the production process, to vary the forms. Doing this, the researcher observes that the form generally represents many practical advantages and is, most frequently, the only possible or the optimal solution of a production problem. By applying this method, it becomes possible to bring to the fore knowledge about the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, pyramids, cylinders, symmetry, etc., that was probably involved in the invention of the production techniques under consideration (see Gerdes, 1985a, 1990c).

Networks, graphs or 'sanddrawings'

One section of Zaslavsky (1973a, 105-109) was dedicated to networks, based on Torday's information (1925) on the Bushongo (actual Zaire)

and Bastin's study (1961) of decorative art of the Tchokwe (Angola) (For educational use of Zaslavsky's analysis of the Bushongo networks, see NCTM (1984) and Whitcombe & Donaldson (1988)). She had not had access to the ethnographical information on such networks published by Baumann (1935, 222-223), Hamelberger (1952) and Dos Santos (1961). Since the publication of **"Africa Counts"** large ethnographical collections of networks have become available: Pearson (1977: 'sandgraphs' observed in the 1920s in the Kwandu-Kuvanga and Muxiku provinces of Angola); Fontinha (1983: '**sona**' or 'sanddrawings' collected principally among the Tchokwe of northeastern Angola during the 1940s and 1950s); and Kubik (1986, 1987a, 1987b, 1988: networks observed among the (Va)luchazi in northwestern Zambia during the 1970s). In order to facilitate the memorisation of their standardised 'sona', the drawing experts used the following mnemonic device. After cleaning and smoothing the ground, they first set out with their fingertips an orthogonal net of equidistant points. Now one or more lines are drawn that 'embrace' the points of the reference frame. By applying their method the drawing experts reduce the memorisation of a whole drawing to that of mostly two numbers (the dimensions of the reference frame) and a geometric algorithm (the rule of how to draw the embracing line(s)). Most drawings belong to a long tradition (cf. Redinha, 1948). They refer to proverbs, fables, games, riddles, animals, etc. and play an important role in the transmission of knowledge and wisdom from one generation to the next. In Kubik's view the 'sona' "transmit empirical mathematical knowledge" (1987a, 450). The geometry of the 'sona' is a "non-euclidean geometry": "The forefathers of the Eastern Angolan peoples discovered higher mathematics and a non-Euclidian geometry on an empirical basis applying their insights to the invention of these unique configurations" (Kubik, 1987b, 108). He calls attention to the symmetry of many 'sona', the implicit rules for construction and rules for anchoring figures of the same type. The ethnographical publications of collections of 'sona' drew the attention of mathematicians. Ascher and Gerdes conducted research on the 'sona', independently one of another. Ascher's study (1988, 1991) deals with geometrical and topological aspects of 'sona', in particular, with symmetries, extension, enlargement through repetition, and isomorphy. Gerdes (1989a, 120-189) analyses symmetry and monolinearity (i.e. a whole figure is made up of only one line) as cultural values, classes of 'sona' and corresponding geometrical algorithms for their construction, systematic construction of monolinear groundpatterns, chain and elimination rules for the construction of monolinear 'sona'. It is suggested that the 'drawing experts' who invented these rules probably knew why they are valid, i.e. they could prove in one or another way the truth of the theorems that these rules express. He advances also with the reconstruction of lost symmetries and monolinearities by means of an analysis of possible drawing errors in reported 'sona' (for an introductory summary of his research findings, see: Gerdes, 1990d, 1991d and 1991e). Inspired by his historical research findings, Gerdes experimented with possibilities to use the 'sona' in mathematics education, in order to value and revive a rich

scientific tradition that had been vanishing (Cf. Gerdes, 1988a, b; 1989a, b, c; 1990a; 1991a and g; cf. Ratteray, 1991). He also initiated a mathematical exploration of the properties of some (extended) classes of 'sona' (see Gerdes, 1989a, 288-297). In a similar way, Kubik's research stimulated a mathematical investigation by Jaritz on a particular class of 'sona' (1983).

Monolinear patterns appear also in other African contexts. For instance, Prussin (1986, 90) displays a symmetric monolinear pattern on a Fulbe warrior's tunic from Senegal. The study of the types and spread of monolinear patterns throughout the African continent deserves further research.

Example of an as yet unexplored area

One of the **many** areas of African culture which have not been studied as yet in view of their inherent mathematical aspects, is that of **string figures**. Elsewhere such an analysis has already begun: De Paula & De Paula (1988) studied the geometry of string figures among the Tapirapé Indians in Brazil; Moore (1986) analysed string figures from the Navaho and other North American Indians and explored their potential for mathematics education. In the case of Africa south of the Sahara, Wedgwood & Schapera's information (1939) on string figures of Botswana may serve as a starting point. They refer to studies on string figures in Central Africa, Liberia, Nigeria, Sierra Leone, South Africa and Tanzania.

In general, once the mathematical character or aspects of cultural elements are recognized, one may try to track the history of the mathematical thinking involved and its (possible) relationships to other cultural-mathematical 'threads' and try to explore their educational and scientific potential.

An example to follow: history, ethnomathematics and educational studies at the Ahmadu-Bello-University

The Faculty of Education of the Ahmadu-Bello-University (Zaria, Nigeria) has been very dynamic in stimulating student research (B.Ed., M.Ed., Ph.D. projects and dissertations) on the mathematical heritage of the peoples of Nigeria, the mathematics used by unschooled children and adults in daily life, and the possibilities to embed this knowledge in mathematics education, e.g. Adaaku (1982: Tiv), Akin & Fapenle (1985: Aweri), Anzenge a.o. (1988: algorithms), Erukoha (1979: Igbo), Gafai (1987: unschooled adults in Katsina State), Henry-Carmichael (1986: unschooled Nupe children), Musa (1987: Hausa), Oladimeji (1987: Yoruba) and Tarbo (1987: Tiv and Idoma unschooled children). For an survey of these research projects, see Shirley (1986, 1988a, 1988b). Ale (1989) does research on the mathematical heritage of the Fulbe (Fulani) and the possibilities to construct a curriculum that builds upon this heritage and fits the needs of the Fulbe (Fulani) people.

If all or most universities become so active as the Ahmadu-Bello-

University, then the knowledge and use of the mathematical traditions south of the Sahara will grow very rapidly and contribute substantially to the valuing of the African heritage.

Interdisciplinarity

As the history of mathematics in Africa may not be considered in isolation either from the development of culture in general, or be dissociated from the evolution of art, cosmology, philosophy, natural sciences, medicine, graphic systems and technology in particular, the historiography of African mathematics has to take into account the research findings from other disciplines:

*** physics, astronomy and cosmology**

Pappademos (1983) presents an outline of Africa's role in the history of physics. Weule (1921) studied early forms of mechanics, based on his fieldwork in eastern Africa. Lynch and Robbins (1983) analyse evidence from Namoratunga, a megalithic site in northwestern Kenya, that suggests that a prehistoric calendar based on detailed astronomical knowledge was in use in eastern Africa (c.300 B.C.). Dundas (1926) describes pre-colonial time-reckoning among the Wachagga (Kilimanjaro-region): the year is divided into twelve months; each month has thirty days and is divided into six periods of five days each. The 'topology of time' among the Iraqw of Tanzania is analysed by Thorton (1980). Concepts of time, time-reckoning and cosmology of the Kagura (eastern Africa), of the Dwala (Cameroon) and of the Tiv (Nigeria) have been described and analysed by Beidelman (1963), Bekombo (1966-1967) and Bohannan (1953) respectively (cf. also Booth, 1975; Kagame, 1976). Lacroix (1972) discusses the time expressions in some west-African languages. Traditional African calendars constitute one of the research themes of the 'Thought Systems in Black Africa' study group in Paris (Baker, 1987, 53). Obenga (1987) reviews the literature on astronomical knowledge in Ancient Egypt, among the Borana (Ethiopia), Dogon, Lobi, Bambara (West Africa), Vili (Congo), Fang (Cameroon, Equatorial Guinee, Gabon), and Mbochi (Congo). Keller's study (1902) deals with the astronomical views of the Isubu in Cameroon.

*** logic and philosophy**

Gluckman (1944), Grootaert (1947), Hegba (1958), Hyombo (1979), Kabasele (1976), Kibasomba (1980) contribute to the analysis of the development of logic in Africa (Cf. also the work of the Inter-African Council for Philosophy [Houtondji, 1987]). Gay and Welmers (1971) analysed mathematics and logic in the Kpelle language (Liberia).

*** technology**

An early tentative attempt to analyse craftwork and technology in eastern Africa in a historical perspective is Stuhlmann's study (1910). Studies of this type are however relatively rare, as Thomas-Emeagwali (1988) stresses: despite the significance of the history of technology there is a relative dearth of writings on the issue in African historiography in

general and in Nigerian historiography in particular. Her evaluation of the role of oral historiography in the reconstruction of the history of technology is important from a methodological point of view. One of the major problems (p.69) with which the historian of technology is confronted in the course of fieldwork is the reluctance of practitioners to divulge technological secrets, so that they can maintain some measure of competitive power over their potential and actual rivals. Today technicians and craftsmen are often unaware of the precise scientific and engineering principles when they use ancient techniques in the making of material objects (iron technology, textile production, basketry, woodwork). Therefore, according to Thomas-Emeagwali, the researcher "must be able to identify the underlying principles at play in the process" (p.70; Cf. Gerdes on 'hidden' mathematical thinking).

*** linguistics and mathematics**

Studies on the multiple and varied interactions between African languages and the learning of mathematics present important data on mathematical concepts and ideas in African cultures. See e.g. the reports of the seminars organized by CASME (1975), UNESCO (1975) and IREM Niamey (1977), and documents such as Yohannes (1974), NUL (1980), and Lassa (1980). Wallman (1965) realized a study on the communication of measurement in Lesotho. Mashinda (1988) analysed logical and linguistic problems met by pupils in Zaire. Zepp (1982a) studied bilinguals' understanding of logical connectives in English and Sesotho and analysed (1983a) inclusive disjunction in West African languages. For an overview, Zepp (1990) may be consulted.

*** developmental psychology and cross-cultural psychology**

Since Gay & Gole's classical study on mathematical learning and mathematical capacities among the Kpelle of Liberia (1967), researchers from the Cornell University and the University of Rochester (USA) have pursued a series of psychological studies on mathematical knowledge and capacities in West Africa: Petitto (1978, 1982: unschooled tailors and merchants in Ivory Coast), Posner (1978, 1982: Baoulé and Dioula children in Ivory Coast), Posner & Baroody (1979: number conservation). In particular Ginsburg (1978) and Petitto & Ginsburg (1982: mental arithmetic) compare their research findings in West-Africa with those obtained in North-America. Etuk (1967) examines Piaget's theory with Yoruba children (Nigeria). Other psychologists showed interest in the cultural context of the understanding of pictorial spatial relationships (pictorial depth perception) and drawing (Deregowski, 1974a, 1976: Kenya, 1980: Zimbabwe [model-making and drawing]; Leach, 1975, 1978: Zimbabwe; Nicholson & Seddon, 1977: Nigeria). Cross-cultural psychologists interested themselves also in symmetry and pattern reproduction (Bentley, 1977: Kenya; Deregowski, 1972, 1974b: Zambia; Jahoda, 1976: Ghana; Tekane, 1963) and in orientation and rotation of drawings (Shapiro, 1960; Serpell, 1971: Zambia).

* **cooperation**

In order to advance with the interdisciplinary study of the history of mathematics in Africa south of the Sahara, it is important to enhance contacts, cooperation and exchange of ideas and research findings between historians of mathematics, mathematics educators, mathematicians, ethnoscientists, ethnographers, etc. and organizations such as the **Studies in the Evolution of Science and Technology in Africa** (SESTA) program of the African Academy of Sciences, the International Centre for Bantu Civilizations (**CICIBA**, Gabon), the Inter-African Council for Philosophy, the International Commission on the History of Mathematics, the International Study Group on the Relations between History and Pedagogy of Mathematics (**HPM**), the International Study Group on Ethnomathematics (**ISGEm**), the International Society for the Interdisciplinary Study of Symmetry (**ISIS-Symmetry**). **AMUCHMA** tries to dynamize this interdisciplinary cooperation.

Relationships between Africa south of the Sahara, North Africa and the outside world

The relationships between the development of mathematics in Africa south of the Sahara and the development of mathematics in Ancient Egypt, in both Hellenistic and Islamic northern Africa, and across the Indian and Atlantic oceans deserve further study.

* **Ancient Egypt**

Many open questions exist. For example, do there exist relationships between the duplication and symmetry patterns in pre-Bantu rock paintings in Mozambique, the 'binary combinatorics' of Pedi augurs in Transvaal (South Africa, cf. Junod, 1935, 559-564), elements of a binary structure in some African numeration systems and in the Ashanti weight measurement (Ghana), the Mbosi duplication (Congo; cf. Obenga, 1973), etc. and the Egyptian duplication method (cf. Gillings, 1972) for multiplication? Another and important open question is: what has been the role of 'Black Africa' in the development of 'classical greek society' (cf. e.g. Bernal's "**Black Athena**", 1987) and its mathematics? Obenga (1991) reanalyses the mathematical documents of Ancient Egypt and stresses their importance in the development of philosophy in Africa. On the basis of mathematical ideas involved in (the invention of) basketry techniques (e.g. woven pyramidal funnels in Mozambique and Zaire) and mat makers (e.g. circular 'spiral' mats) and taking into account the cultural linkages between ancient Egypt and the rest of 'Black Africa' (cf. Diop, 1981), it is possible to formulate new hypotheses on how the ancient Egyptian formulas for the area of a circle (Gerdes, 1985b) and for the volume of a truncated pyramid (Gerdes, 1985a, 1990c) may have been found.

* **The Islamic-Arabic world** (cf. Djebbar, 1992)

Throughout history there have been many and varied contacts between Africa south of the Sahara and North Africa. Since the birth and spread of Islam, relations have been intensified and/or extended. Zaslavsky (1973a, 138-151) discusses the work of Muhammed ibn Muhammed from Katsina (now northern Nigeria) on chronograms and magic squares. Muhammed ibn Muhammed, who had been a pupil of Muhammed Alwali of Bagirmi, made a pilgrimage to Mecca in 1730 and he died in Cairo in 1741. At the 2nd Pan-African Congress of Mathematicians, Kani (Ahmadu-Bello-University, Zaria) presented a paper on 'Ilm al-Hisab' (arithmetics) as studied by Muslim scholars of pre-colonial northern Nigeria and discussed the work of the same Muhammed ibn Muhammed al Katsinawi on magic squares and numerological patterns. Recently a manuscript of his has been found in Marrakesh (Morocco) (Djebbar, 1990, oral communication). Prussin (1986, p.76, 147) refers to the use of magic squares in amulets among the Fulbe, and in Niger, Benin and Timbuktu (Mali). Thomas-Emeagwali (1987) reflects on the development of science in the Islamic world and its diffusion into Nigeria before 1903. Diop (1960, 167) refers to the study of formal logic in Timbuktu and Lapousterle (Bamako, Mali) prepares a study on the contents of three mathematical manuscripts, written in Arabic, that belong to the Ahmad Baba Library in Timbuktu. One of the three manuscripts, whose calligraphy is typical for Africa south of the Sahara, seems to have been written by a mathematician from Mali, al-Arwani. The other two contain references to medieval mathematicians from the Maghreb. Systematic search in libraries and archives will probably lead to the discovery of more mathematical manuscripts from Muslim scholars south of the Sahara. There may also exist other sources, also written in Arabic, on mathematical activities south of the Sahara, e.g. on the eastern coast of Africa, like the comments of traveller Ahmad Ibn-Madjid at the end of the 15th century on counting of the Wac-Wac, probably not a Bantu but a Khoi-San people, living at that time in southern and central Mozambique (cf. Chumovsky & Jirmounsky, 1957, R.94 v.18).

* **Across the oceans**

The possible mutual links between the development of mathematics across the Indian Ocean still remain to be studied (cf. UNESCO, 1980, for historical relations across the Indian Ocean). E.g. what the possible influence of mathematical ideas from slaves from the African continent and from India and Indonesia on the development of (ethno)-mathematics in Madagascar, Mauritius and other islands has been. What mathematics has been brought to the Americas by the slaves? Which mathematical ideas have survived in one way or another? 'Mancala' and maybe other games with mathematical 'ingredients' are played in the Caribbean and may be compared with their 'ancestors' in Africa. Ferreira (1982, 2) refers to a study by one of his students on

geometric symbology in 'Ubunda' and 'Candomblé' in Brazil. This research area remains almost virgin.

In Fauvel & Gerdes' paper (1990) on Thomas Fuller (1710-1790), the African slave and calculating prodigy, shipped to America in 1724, it is suggested that ethno-mathematical research may complement the analysis of written sources. Fuller's exceptional abilities cannot be understood except through closer examination of the cultural context that stimulated their development. Like the professional knowledge of the Tchokwe smiths, the knowledge of the Tchokwe drawing experts (*akwa kuta sona*) was mostly secret, transmitted from father to son. When a blacksmith or a drawing expert was taken prisoner and sold as a slave, his specific professional knowledge might disappear completely from his village or region. Thus the slave trade was extremely destructive of the development of the existing mathematical traditions and potential, through breaking the professional continuity and depriving Africa of bearers of mathematical knowledge and skill such as Thomas Fuller. Recent ethnomathematical research in Nigeria, as summarized by Shirley (1988), shows the survival, nonetheless, of a rich tradition of mental calculations among illiterate people. It would be interesting and valuable to search for new records with data on the geographical or ethnic origin of Thomas Fuller, and to correlate this information with ethnomathematical and historical research on the same region or ethnic group. On the other hand, it would be interesting and important to analyse the colonial impact on 'professional' knowledge of specialists, as in the case of the Tchokwe drawing experts who belonged to a social elite, in contrast to its impact on less monopolized knowledge such as that underlying many basket weaving techniques.

One possible way in which the slave trade influenced the development of arithmetical knowledge in Africa has been described by Clarkson (1788) and Scripture (1891): "Perhaps brought to the front or produced by the necessity of competing with English traders armed with pencil and paper, many of the old-time slave-dealers of Africa seem to have been ready reckoners, and that, too, for a practical purpose...The shipcaptains are said to have complained that it became more and more difficult to make good bargains with such sharp arithmeticians" (cited in AMUCHMA 3, 4). It would be interesting to explore further this and other possible influences, such as the disappearance or undermining of traditional African mathematical education by the physical elimination or 'exportation' of the bearers of mathematical knowledge. The destructive impact of colonialism and the slave trade on Africa is one of the reasons indicated by Njock (1985) that explain why Africa has not known such a spectacular development of mathematics in the last five centuries, as Europe has known. Other reasons he gives deal with the geography of the continent (migratory movement) and wars. This also constitutes an important research area that deserves further study.

In AMUCHMA 3 (1989) it has been suggested that it would be interesting to search African literature, including autobiographies (cf. Fataki, 1991; Mugambi, 1991), for information on mathematics education in the colonial period and the reaction to it (e.g. attitudes towards mathematics

and mathematics teachers). A systematic analysis of the ideas (and prejudices) of missionaries, colonial administrators and educators about mathematical capacities of their African 'subjects' (e.g. the comments of Junod on the 'lack of mathematical ability' in Mozambique (Junod, 1935, 151-155, 576-577) and of their implications, might be worthwhile and revealing for the new generations.

Recent history

The impact of (neo)colonisation, with its subsequent 'transplantation of curricula', on mathematics education and the recent history of mathematics education in Africa has been the subject of diverse publications: Tanzania (Pythiam, 1971; Mmari, 1978, 1980, 1991; Seka, 1987); Nigeria (Ohuche, 1978; Shirley, 1980); Sierra Leone (Williams, 1978); Sudan (El Sawi, 1978; El Tom, 1983); Mozambique (Gerdes, 1980a, 1980b, 1981; Draisma, 1985, 1986, Cherinda, 1991); Swaziland (Masinga, 1987); Uganda (Timkumanya, 1989). Gerdes (1984) reproduces the autobiographies of the winners of the first mathematics olympiads in Mozambique. Navez (University of Burundi) does research on the evolution of mathematics curricula at secondary school level in Burundi.

An interesting theme in the recent history of mathematics education in Africa, that seems to deserve study, is that of the emergence and evolution of continental and regional mathematics curriculum development projects, like the African Mathematics Program (AMP), School Mathematics Project for East Africa (SMPEA), Joint Mathematics Project, East African Regional Mathematics Program (EARMP), and West African Regional Mathematics Program (WARMP). The history of mathematical associations and journals, of mathematical departments and schools, are other possible research topics.

The introduction and spread in Africa south of the Sahara of new mathematical research areas and related fields such as statistics, informatics, and computer science in relationship to the introduction and spread of new technologies, like the use of computers, has not been studied as yet, nor the implications for the countries involved.

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Paulus Gerdes

ON THE HISTORY OF MATHEMATICS IN SUBSAHARAN AFRICA*

* This paper constitutes the 2nd part of the study "Recent research on the History of Mathematics in Africa: an overview" by A.Djebbar and P.Gerdes

Introduction

Why study the history of mathematics in Africa south of the Sahara?

Broad conception of 'history' and 'mathematics'

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Games

Geometry and architecture

Uncovering 'hidden' mathematical ideas: geometrical form

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Relationships between Africa south of the Sahara, north Africa and the outside world

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