SOLUTION FOR HOMEWORK #9

1. Section 11.3

6. The function $f(x) = e^{-x}$ is continuous, positive, and decreasing on $[1, \infty)$, so we can apply the integral test.

$$\int_1^\infty e^{-x}\,dx = \lim_{t\to\infty}\int_1^t e^{-x}\,dx = \lim_{t\to\infty}(-e^{-x})|_1^t = \lim_{t\to\infty}(-e^{-t}+e^{-1}) = e^{-1},$$

thus $\sum_{n=1}^\infty e^{-n}$ converges.

10. Both $\sum_{n=1}^\infty n^{-1.4}$ and $\sum_{n=1}^\infty n^{-1.2}$ are p-series with $p > 1$, so they converge. Consequently, $\sum_{n=1}^\infty(n^{-1.4} + 3n^{-1.2})$ converges.

26. One has $(x\ln x[\ln(\ln x)]^p)' = (p + (\ln x - 1)\ln\ln x)(\ln\ln x)^{p-1} > 0$ when $x \geq M$ for some constant $M > 3$. Therefore the function $f(x) = \frac{1}{x\ln x[\ln(\ln x)]^p}$ is continuous, positive, and decreasing on $[M, \infty)$, and so we may apply the integral test. When $p \neq 1$,

$$\int_3^\infty \frac{1}{x\ln x[\ln(\ln x)]^p}\,dx = \lim_{t\to\infty}(\frac{(\ln\ln x)^{-p+1}}{-p+1})|_3^t$$

converges exactly when $-p+1 < 0 \Leftrightarrow p > 1$. When $p = 1$,

$$\int_3^\infty \frac{1}{x\ln x[\ln(\ln x)]}\,dx = \lim_{t\to\infty}(\ln\ln\ln x)|_3^t$$

diverges to ∞. Thus the series converges exactly when $p > 1$.

2. Section 10.4

4. $\frac{2}{n^3+4} < \frac{2}{n^3}$ for all $n \geq 1$. The series $\sum_{n=1}^\infty \frac{2}{n^3}$ converges since it is a constant multiple of a convergent p-series. Thus the series $\sum_{n=1}^\infty \frac{2}{n^3+4}$ converges by the comparisen test.

6. $\frac{1}{n-\sqrt{n}} > \frac{1}{n}$ for all $n \geq 2$, thus $\sum_{n=2}^\infty \frac{1}{n-\sqrt{n}}$ converges via comparison with the divergent series $\sum_{n=2}^\infty \frac{1}{n}$.

10. $\lim_{n\to\infty}(\frac{n^2-1}{3n^4+1})/(\frac{1}{n^2}) = \frac{1}{3} > 0$. Since $\sum_{n=1}^\infty \frac{1}{n^2}$ is a convergent p-series, $\sum_{n=1}^\infty \frac{n^2-1}{3n^4+1}$ converges by the limit comparison test.