SOLUTION FOR HOMEWORK #8

1. Section 11.1

16. $a_n = \frac{1+1/n}{3-1/n} \to \frac{1}{3}$ as $n \to \infty$.

20. $a_n = \frac{\sqrt{n}}{1/\sqrt{n}+1}$ diverges to ∞ as $n \to \infty$.

24. As $n \to \infty$, $2/n \to 0$, so $a_n = \cos(2/n) \to 1$.

28. $\frac{\ln n}{\ln 2n} = \frac{\ln n}{\ln 2+\ln n} = \frac{1}{\frac{\ln 2}{\ln n}+1} \to 1$ as $n \to \infty$.

32. $a_n = \ln \frac{n+1}{n} \to \ln 1 = 0$ as $n \to \infty$.

36. $|a_n| = \frac{|\sin 2n|}{1+\sqrt{n}} \le \frac{1}{1+\sqrt{n}}$. Since $\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0$, $\lim_{n\to\infty} |a_n| = 0$ by the squeeze theorem. Thus $\lim_{n\to\infty} a_n = 0$.

62. (a). Note that $a_1 \le 2$. Suppose that $a_n \le 2$. Then $a_{n+1} = \sqrt{2+a_n} \le \sqrt{2+2} = 2$. Thus $a_n \le 2$ for all $n \ge 1$ by induction. Then $a_{n+1} - a_n = \sqrt{2+a_n} - a_n = \frac{2+a_n-a_n^2}{\sqrt{2+a_n}+a_n} = \frac{(2-a_n)(1+a_n)}{\sqrt{2+a_n}+a_n} \ge 0$ for all $n \ge 1$. In other words, $\{a_n\}$ is increasing. Therefore $\lim_{n\to\infty} a_n$ exists.

(b). Set $L = \lim_{n\to\infty} a_n$. Taking limits on both sides of $a_{n+1} = \sqrt{2+a_n}$ we get $L = \sqrt{2+L}$. Thus $L^2 = 2+L$, and hence $L = 2$. So $\lim_{n\to\infty} a_n = 2$.

2. Section 11.2

12. $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$ is a geometric series with $r = -2$. Since $|r| = 2 > 1$, the series diverges.

18. The series $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ is a geometric series with $r = \frac{1}{\sqrt{2}}$. Since $|r| = \frac{1}{\sqrt{2}} < 1$, the series converges to $\frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1} = \sqrt{2}(\sqrt{2}+1) = 2+\sqrt{2}$.

20. The series $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is a geometric series with $r = \frac{e}{3}$. Since $|r| = \frac{e}{3} < 1$, it converges to $\frac{e}{1-\frac{e}{3}} = \frac{3e}{3-e}$.

24. Since $\lim_{n\to\infty} \frac{(n+1)^2}{n(n+2)} = 1 \ne 0$, the series $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$ diverges.

34. The series $\sum_{n=1}^{\infty} \frac{3}{5^n}$ is a geometric series with $|r| = \frac{1}{5}$ and hence converges. The series $\sum_{n=1}^{\infty} \frac{2}{n}$ diverges. Thus the series $\sum_{n=1}^{\infty}(\frac{3}{5^n} + \frac{2}{n})$ diverges.

36. $0.\overline{73} = \sum_{n=0}^{\infty} \frac{73}{100} \cdot \frac{1}{100^n} = \frac{73/100}{1-\frac{1}{100}} = \frac{73}{99}$.