1. Section 11.1

16. \(a_n = \frac{1+1/n}{3-1/n} \to \frac{1}{3} \) as \(n \to \infty\).

20. \(a_n = \frac{\sqrt{n}}{1/\sqrt{n}+1}\) diverges to \(\infty\) as \(n \to \infty\).

24. As \(n \to \infty\), \(2/n \to 0\), so \(a_n = \cos(2/n) \to 1\).

28. \(\ln n / \ln 2 = \ln n \ln 2 + \ln n = 1 \ln 2 + \ln n \to 1\) as \(n \to \infty\).

32. \(a_n = \ln n + 1 \to \ln 1 = 0\) as \(n \to \infty\).

36. \(|a_n| = \left|\frac{\sin 2n}{1+\sqrt{n}}\right| \leq \frac{1}{1+\sqrt{n}}\). Since \(\lim_{n \to \infty} \frac{1}{1+\sqrt{n}} = 0\), \(\lim_{n \to \infty} |a_n| = 0\) by the squeeze theorem. Thus \(\lim_{n \to \infty} a_n = 0\).

62. (a). Note that \(a_1 \leq 2\). Suppose that \(a_n \leq 2\). Then \(a_{n+1} = \sqrt{2+a_n} \leq \sqrt{2+2} = 2\). Thus \(a_n \leq 2\) for all \(n \geq 1\) by induction. Then \(a_{n+1} - a_n = \sqrt{2+a_n} - a_n = \frac{2+a_n-a_n^2}{\sqrt{2+a_n}+a_n} = \frac{(2-a_n)(1+a_n)}{\sqrt{2+a_n}+a_n} \geq 0\) for all \(n \geq 1\). In other words, \(\{a_n\}\) is increasing. Therefore \(\lim_{n \to \infty} a_n\) exists.

(b). Set \(L = \lim_{n \to \infty} a_n\). Taking limits on both sides of \(a_{n+1} = \sqrt{2+a_n}\) we get \(L = \sqrt{2+L}\). Thus \(L^2 = 2 + L\), and hence \(L = 2\). So \(\lim_{n \to \infty} a_n = 2\).

2. Section 11.2

12. \(\frac{1}{5} - \frac{1}{4} + \frac{1}{3} - 1 + \cdots\) is a geometric series with \(r = -2\). Since \(|r| = 2 > 1\), the series diverges.

18. The series \(\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}\) is a geometric series with \(r = \frac{1}{\sqrt{2}}\). Since \(|r| = \frac{1}{\sqrt{2}} < 1\), the series converges to \(\frac{1}{1-\sqrt{2}/2} = \frac{\sqrt{2}}{\sqrt{2}-1} = \sqrt{2}(\sqrt{2}+1) = 2 + \sqrt{2}\).

20. The series \(\sum_{n=1}^{\infty} \frac{e^n}{3^n-1}\) is a geometric series with \(r = \frac{e}{3}\). Since \(|r| = \frac{e}{3} < 1\), it converges to \(\frac{\frac{e}{2}}{1-\frac{e}{3}} = \frac{3e}{3-e}\).

24. Since \(\lim_{n \to \infty} \frac{(n+1)^2}{n(n+2)} = 1 \neq 0\), the series \(\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}\) diverges.
34. The series $\sum_{n=1}^{\infty} \frac{3}{5^n}$ is a geometric series with $|r| = \frac{1}{5}$ and hence converges. The series $\sum_{n=1}^{\infty} \frac{2}{n}$ diverges. Thus the series $\sum_{n=1}^{\infty} \left( \frac{3}{5^n} + \frac{2}{n} \right)$ diverges.

36. $0.73 = \sum_{n=0}^{\infty} \frac{73}{100} \cdot \frac{1}{100^n} = \frac{73/100}{1 - \frac{1}{100}} = \frac{73}{99}$. 