SOLUTION FOR HOMEWORK #7

1. Section 8.2

6. We may just use the part of the curve above the x-axis, given by $y = 3\sqrt{x-2}$. So $dy/dx = \frac{3}{2\sqrt{x-2}}$ and $1+(dy/dx)^2 = 1+\frac{9}{4(x-2)} = \frac{4x+1}{4(x-2)}$. Thus

$$\begin{aligned} S &= \int_2^6 2\pi \cdot 3\sqrt{x-2}\sqrt{\frac{4x+1}{4(x-2)}}\,dx = 6\pi\int_2^6 \sqrt{x+\frac{1}{4}}\,dx \\ &= 6\pi \cdot \frac{2}{3}(x+\frac{1}{4})^{3/2}|_2^6 = 4\pi(\frac{125}{8} - \frac{27}{8}) = 49\pi. \end{aligned}$$

10. $dy/dx = \frac{x^2}{2} - \frac{1}{2x^2}$ and $1+(dy/dx)^2 = 1+(\frac{x^2}{2}-\frac{1}{2x^2})^2 = (\frac{x^2}{2}+\frac{1}{2x^2})^2$. Thus

$$\begin{aligned} S &= \int_{1/2}^1 2\pi(\frac{x^3}{6}+\frac{1}{2x})\sqrt{(\frac{x^2}{2}+\frac{1}{2x^2})^2}\,dx = 2\pi\int_{1/2}^1 (\frac{x^3}{6}+\frac{1}{2x})(\frac{x^2}{2}+\frac{1}{2x^2})\,dx \\ &= 2\pi\int_{1/2}^1 (\frac{x^5}{12}+\frac{x}{3}+\frac{1}{4x^3})\,dx = 2\pi(\frac{x^6}{72}+\frac{x^2}{6}-\frac{x^{-2}}{8})|_{1/2}^1 \\ &= 2\pi((\frac{1}{72}+\frac{1}{6}-\frac{1}{8}) - (\frac{1}{64\cdot 72}+\frac{1}{24}-\frac{1}{2})) = \frac{263}{256}\pi. \end{aligned}$$

14. $1+(dy/dx)^2 = 1+4x^2$. Thus

$$S = \int_0^1 2\pi x\sqrt{1+4x^2}\,dx \overset{u=1+4x^2}{=} \int_1^5 \frac{\pi}{4}\sqrt{u}\,du = \frac{\pi}{4}\cdot\frac{2}{3}u^{3/2}|_1^5 = \frac{\pi}{6}(5\sqrt{5}-1).$$

2. Section 10.2

4. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t^2-1}{4t}$. When $t=3$, $(x,y) = (19,6)$ and $\frac{dy}{dx} = \frac{2}{3}$. So $y-6 = \frac{2}{3}(x-19)$ is an equation of the tangent line.

12. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-12}$. So

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{dx/dt} = \frac{\frac{d}{dt}(\frac{2t}{3t^2-12})}{3t^2-12} = \frac{\frac{-6t^2-24}{(3t^2-12)^2}}{3t^2-12} = \frac{-2(t^2+4)}{9(t^2-4)^3}.$$

Thus the curve is concave upwrad when $t^2-4<0 \Leftrightarrow -2<t<2$.

42. $dx/d\theta = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\theta\cos\theta$ and $dy/d\theta = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\theta\sin\theta$. So $(dx/d\theta)^2 + (dy/d\theta)^2 = a^2(\theta^2\cos^2\theta + \theta^2\sin^2\theta) = a^2\theta^2$. Thus

$$L = \int_0^\pi \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta = \int_0^\pi a\theta\, d\theta = \frac{a}{2}\theta^2|_0^\pi = \frac{a}{2}\pi^2.$$

60. $(dx/dt)^2 + (dy/dt)^2 = (3-3t^2)^2 + (6t)^2 = (3(1+t^2))^2$. So

$$\begin{aligned} S &= \int_0^1 2\pi \cdot 3t^2 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = 6\pi \int_0^1 t^2 \cdot 3(1+t^2)\, dt \\ &= 18\pi(\frac{1}{3}t^3 + \frac{1}{5}t^5)|_0^1 = \frac{48}{5}\pi. \end{aligned}$$