SOLUTION FOR HOMEWORK #6

1. Section 7.8

8.

$$\begin{aligned}\int_0^\infty \frac{x}{(x^2+2)^2}\,dx &= \lim_{t\to\infty}\int_0^t \frac{x}{(x^2+2)^2}\,dx = \lim_{t\to\infty}\frac{-1}{2(x^2+2)}\Big|_0^t \\ &= \lim_{t\to\infty}\left(\frac{-1}{2(t^2+2)} - \frac{-1}{4}\right) = \frac{1}{4}.\end{aligned}$$

The improper integral is convergent.

14.

$$\int_{-\infty}^\infty x^2 e^{-x^3}\,dx = \int_0^\infty x^2 e^{-x^3}\,dx + \int_{-\infty}^0 x^2 e^{-x^3}\,dx,$$

and

$$\int_{-\infty}^0 x^2 e^{-x^3}\,dx = \lim_{t\to-\infty}\int_t^0 x^2 e^{-x^3}\,dx = \lim_{t\to-\infty}\frac{-e^{-x^3}}{3}\Big|_t^0 = \lim_{t\to-\infty}\frac{-1+e^{-t^3}}{3} = \infty.$$

The improper integral is divergent.

20.

$$\int re^{r/3}\,dr = 3re^{r/3} - \int 3e^{r/3}\,dr = 3re^{r/3} - 9e^{r/3} + C.$$

So

$$\begin{aligned}\int_{-\infty}^6 re^{r/3}\,dr &= \lim_{t\to-\infty}\int_t^6 re^{r/3}\,dr = \lim_{t\to-\infty}(3re^{r/3} - 9e^{r/3})\Big|_t^6 \\ &= \lim_{t\to-\infty}[(18e^2 - 9e^2) - (3te^{t/3} - 9e^{t/3})] = 9e^2.\end{aligned}$$

The improper integral is convergent.

28.

$$\int_0^3 \frac{1}{x\sqrt{x}}\,dx = \lim_{t\to0^+}\int_t^3 \frac{1}{x\sqrt{x}}\,dx = \lim_{t\to0^+}(-2x^{-1/2})\Big|_t^3 = \lim_{t\to0^+}(-2\cdot3^{-1/2}+2t^{-1/2}) = \infty.$$

The improper integral is divergent.

30.

$$\int_1^9 \frac{1}{\sqrt[3]{x-9}}\,dx = \lim_{t\to 9^-}\int_1^t \frac{1}{\sqrt[3]{x-9}}\,dx = \lim_{t\to 9^-}\frac{3}{2}(x-9)^{2/3}|_1^t = \lim_{t\to 9^-}\frac{3}{2}[(t-9)^{2/3}-4] = -6.$$

The improper integral is convergent.

50. For $x \geq 1$, $\frac{2+e^{-x}}{x} \geq \frac{2}{x} \geq \frac{1}{x}$. Since $\int_1^\infty \frac{1}{x}\,dx$ diverges, $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ diverges by the comparison test.

2. Section 8.1

6. Since $y > 0$, $y = 2(x+4)^{3/2}$ and hence $dy/dx = 3(x+4)^{1/2}$. So the length

$$L = \int_0^2 \sqrt{1+(3(x+4)^{1/2})^2}\,dx = \int_0^2 \sqrt{9x+37}\,dx = \frac{2}{27}(9x+37)^{3/2}|_0^2 = \frac{2}{27}(55^{3/2}-37^{3/2}).$$

8. $dy/dx = x - \frac{1}{4x}$. So the length

$$\begin{aligned} L &= \int_2^4 \sqrt{1+(x-\frac{1}{4x})^2}\,dx = \int_2^4 \sqrt{(x+\frac{1}{4x})^2}\,dx = \int_2^4 (x+\frac{1}{4x})\,dx \\ &= (\frac{x^2}{2}+\frac{\ln|x|}{4})|_2^4 = (8+\frac{\ln 4}{4}) - (2+\frac{\ln 2}{4}) = 6+\frac{\ln 2}{4}. \end{aligned}$$