SOLUTION FOR HOMEWORK #5

1. Section 7.5

4. Let $u = x^2$. Then $du = 2x\,dx$ and

$$\int \frac{x}{\sqrt{3-x^4}}\,dx = \int \frac{1}{\sqrt{3-u^2}}\frac{du}{2} = \frac{1}{2}\sin^{-1}\frac{u}{\sqrt{3}} + C,$$

where C is a constant.

10. Let $u = x^2$. Then $du = 2x\,dx$ and

$$\begin{aligned}\int \frac{x}{x^4+x^2+1}\,dx &= \frac{1}{2}\int \frac{du}{u^2+u+1} = \frac{1}{2}\int \frac{du}{(u+1/2)^2+3/4}\\ &\overset{v=u+1/2}{=} \frac{1}{2}\int \frac{dv}{v^2+3/4} = \frac{1}{2}\cdot\frac{1}{\sqrt{3/4}}\tan^{-1}\frac{v}{\sqrt{3/4}} + C\\ &= \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x^2+1}{\sqrt{3}} + C,\end{aligned}$$

where C is a constant.

12. Let $u = \cos x$. Then $du = -\sin x\,dx$ and

$$\int \sin x\cos(\cos x)\,dx = -\int \cos u\,du = -\sin u + C = -\sin(\cos x) + C,$$

where C is a constant.

14. Let $u = \ln x$. Then $du = \frac{1}{x}\,dx$ and

$$\begin{aligned}\int \frac{\sqrt{1+\ln x}}{x\ln x}\,dx &= \int \frac{\sqrt{1+u}}{u}\,du\\ &\overset{v=\sqrt{1+u}}{=} \int \frac{v}{v^2-1}2v\,dv = 2\int (1+\frac{1}{v^2-1})\,dv\\ &= 2\int (1+\frac{1}{2}\frac{1}{v-1} - \frac{1}{2}\frac{1}{v+1})\,dv\\ &= 2v + \ln|v-1| - \ln|v+1| + C\\ &= 2\sqrt{1+\ln x} + \ln|\sqrt{1+\ln x}-1| - \ln(\sqrt{1+\ln x}+1) + C,\end{aligned}$$

where C is a constant.

20. Let $u = \sqrt[3]{x}$. Then $x = u^3$ and $dx = 3u^2\,du$. Thus

$$\begin{aligned}\int e^{\sqrt[3]{x}}\,dx &= 3\int u^2e^u\,du = 3(u^2e^u - \int (2u)e^u\,du)\\ &= 3(u^2e^u - (2ue^u - \int 2e^u\,du)) = 3(u^2e^u - 2ue^u + 2e^u) + C\\ &= 3e^{\sqrt[3]{x}}(x^{2/3} - 2x^{1/3} + 2) + C,\end{aligned}$$

where C is a constant.

2. Section 7.6

2.

$$\begin{aligned}\int \frac{3x}{\sqrt{3-2x}}\,dx &= 3\int \frac{x}{\sqrt{3+(-2)x}}\,dx\\ &\overset{55}{=} 3(\frac{2}{3(-2)^2}((-2)x - 2\cdot 3)\sqrt{3+(-2)x}) + C\\ &= -(x+3)\sqrt{3-2x} + C,\end{aligned}$$

where C is a constant.

4.

$$\int e^{2\theta}\sin 3\theta\,d\theta \overset{98}{=} \frac{e^{2\theta}}{2^2+3^2}(2\sin 3\theta - 3\cos 3\theta) + C = \frac{e^{2\theta}}{13}(2\sin 3\theta - 3\cos 3\theta) + C,$$

where C is a constant.