SOLUTION FOR HOMEWORK #3

1. Section 7.1

4. Let $f = x, g' = e^{-x}$. Then $f' = 1, g = -e^{-x}$. Thus

$$\int xe^{-x}dx = -xe^{-x} - \int 1 \cdot (-e^{-x})dx = -xe^{-x} - e^{-x} + C,$$

where C is a constant.

6. Let $f = t, g' = \sin 2t$. Then $f' = 1, g = -\frac{1}{2}\cos 2t$. Thus

$$\int t\sin 2tdt = -\frac{1}{2}t\cos 2t - \int 1 \cdot (-\frac{1}{2}\cos 2t)dt = -\frac{1}{2}t\cos 2t + \frac{1}{4}\sin 2t + C,$$

where C is a constant.

8. Let $f = x^2, g' = \cos mx$. Then $f' = 2x, g = \frac{1}{m}\sin mx$. Thus

$$\int x^2\cos mxdx = \frac{1}{m}x^2\sin mx - \frac{2}{m}\int x\sin mxdx.$$

Next let $F = x, G' = \sin mx$. Then $F' = 1, G = -\frac{1}{m}\cos mx$. Thus

$$\int x\sin mxdx = -\frac{1}{m}x\cos mx - \int 1 \cdot (-\frac{1}{m}\cos mx)dx = -\frac{1}{m}x\cos mx + \frac{1}{m^2}\sin mx + C,$$

where C is a constant. Putting these two equations together, we get

$$\int x^2\cos mxdx = \frac{1}{m}x^2\sin mx + \frac{2}{m^2}x\cos mx - \frac{2}{m^3}\sin mx + C',$$

where C' is a constant.

12. Let $f = \ln p, g' = p^5$. Then $f' = \frac{1}{p}, g = \frac{1}{6}p^6$. Thus

$$\int p^5\ln pdp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5dp = \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C,$$

where C is a constant.

14. Let $f = t^3, g' = e^t$. Then $f' = 3t^2, g = e^t$. Thus $\int t^3 e^t dt = t^3 e^t - \int 3t^2 e^t dt$. Integrating by parts twice more we get

$$\begin{aligned}\int t^3 e^t dt &= t^3 e^t - (3t^2 e^t - \int 6te^t dt) = t^3 e^t - 3t^2 e^t + 6te^t - \int 6e^t dt \\ &= t^3 e^t - 3t^2 e^t + 6te^t - 6e^t + C,\end{aligned}$$

where C is a constant.

2. Section 7.2

2.

$$\begin{aligned}\int \sin^6 x \cos^3 x dx &= \int \sin^6 x \cos^2 x \cos x dx = \int \sin^6 x (1 - \sin^2 x) \cos x dx \\ &\overset{u=\sin x}{=} \int u^6 (1 - u^2) du = \int (u^6 - u^8) du = \frac{1}{7} u^7 - \frac{1}{9} u^9 + C \\ &= \frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C,\end{aligned}$$

where C is a constant.

10.

$$\begin{aligned}\int \cos^6 \theta d\theta &= \int (\frac{1}{2}(1 + \cos 2\theta))^3 d\theta = \frac{1}{8} \int (1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta) d\theta \\ &= \frac{1}{8}(\theta + \frac{3}{2}\sin 2\theta) + \frac{1}{8}\int \frac{3}{2}(1 + \cos 4\theta) d\theta + \frac{1}{8}\int ((1 - \sin^2 2\theta)\cos 2\theta) d\theta \\ &\overset{u=\sin 2\theta}{=} \frac{1}{8}(\theta + \frac{3}{2}\sin 2\theta) + \frac{3}{16}(\theta + \frac{1}{4}\sin 4\theta) + \frac{1}{8}\int (1 - u^2)(\frac{1}{2} du) \\ &= \frac{1}{8}(\theta + \frac{3}{2}\sin 2\theta) + \frac{3}{16}(\theta + \frac{1}{4}\sin 4\theta) + \frac{1}{16}(u - \frac{1}{3}u^3) + C \\ &= \frac{1}{8}(\theta + \frac{3}{2}\sin 2\theta) + \frac{3}{16}(\theta + \frac{1}{4}\sin 4\theta) + \frac{1}{16}(\sin 2\theta - \frac{1}{3}\sin^3 2\theta) + C \\ &= \frac{5}{16}\theta + \frac{1}{4}\sin 2\theta - \frac{1}{48}\sin^3 2\theta + \frac{3}{64}\sin 4\theta + C,\end{aligned}$$

where C is a constant. Thus

$$\begin{aligned}\int_0^\pi \cos^6 \theta d\theta &= (\frac{5}{16}\theta + \frac{1}{4}\sin 2\theta - \frac{1}{48}\sin^3 2\theta + \frac{3}{64}\sin 4\theta)|_0^\pi \\ &= \frac{5}{16}\pi.\end{aligned}$$

28.

$$
\begin{aligned}
\tan^3(2x)\sec^5(2x)dx &= \int \tan^2(2x)\sec^4(2x)\cdot\sec(2x)\tan(2x)dx \\
&\overset{u=\sec 2x}{=} \int (u^2-1)u^4(\frac{1}{2}du) \\
&= \frac{1}{2}\int (u^6-u^4)du = \frac{1}{14}u^7 - \frac{1}{10}u^5 + C \\
&= \frac{1}{14}\sec^7(2x) - \frac{1}{10}\sec^5(2x) + C,
\end{aligned}
$$

where C is a constant.

42.

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\begin{aligned}
\int \sin 3x \cos x dx &= \int \frac{1}{2}(\sin(3x+x)+\sin(3x-x))dx = \frac{1}{2}\int(\sin 4x + \sin 2x)dx \\
&= -\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C,
\end{aligned}
$$

where C is a constant.