SOLUTION FOR HOMEWORK #2

1. Section 6.2

2. A cross-section is a disk with radius e^x. So it has area $A(x) = \pi(e^x)^2$.

$$V = \int_0^1 A(x)dx = \int_0^1 \pi(e^x)^2 dx = \pi \int_0^1 e^{2x} dx = \frac{1}{2}\pi(e^{2x})|_0^1 = \frac{\pi}{2}(e^2 - 1).$$

6. A cross-section is a disk with radius $y - y^2$. So it has area $A(y) = \pi(y - y^2)^2$.

$$\begin{aligned} V &= \int_0^1 A(y)dy = \int_0^1 \pi(y - y^2)^2 dy = \pi \int_0^1 (y^4 - 2y^3 + y^2)dy \\ &= \pi(\frac{1}{5}y^5 - \frac{1}{2}y^4 + \frac{1}{3}y^3)|_0^1 = \pi(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}) = \frac{\pi}{30}. \end{aligned}$$

12. A cross-section is a disk with radius $4 - x^2$. So it has area $A(x) = \pi(4 - x^2)^2$.

$$\begin{aligned} V &= \int_{-2}^2 A(x)dx = 2\int_0^2 A(x)dx = 2\int_0^2 \pi(4 - x^2)^2 dx \\ &= 2\pi \int_0^2 (x^4 - 8x^2 + 16)dx = 2\pi(\frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x)|_0^2 \\ &= 2\pi(\frac{32}{5} - \frac{64}{3} + 32) = 64\pi(\frac{1}{5} - \frac{2}{3} + 1) = 64\pi \cdot \frac{8}{15} = \frac{512\pi}{15}. \end{aligned}$$

2. Section 6.3

4.

$$\begin{aligned} V &= \int_0^1 2\pi x \cdot x^2 dx = 2\pi \int_0^1 x^3 dx \\ &= 2\pi(\frac{1}{4}x^4)|_0^1 = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}. \end{aligned}$$

6.

$$\begin{aligned} V &= 2\pi \int_0^3 (x((3 + 2x - x^2) - (3 - x)))dx = 2\pi \int_0^3 (3x^2 - x^3)dx \\ &= 2\pi(x^3 - \frac{1}{4}x^4)|_0^3 = 2\pi(27 - \frac{81}{4}) = 2\pi(\frac{27}{4}) = \frac{27\pi}{2}. \end{aligned}$$

18.

$$\begin{aligned} V &= \int_0^4 2\pi(x-(-2))((8x-2x^2)-(4x-x^2))dx = 2\pi\int_0^4(-x^3+2x^2+8x)dx \\ &= 2\pi(-\frac{1}{4}x^4+\frac{2}{3}x^3+4x^2)|_0^4 = 2\pi(-64+\frac{128}{3}+64) = \frac{256}{3}\pi. \end{aligned}$$

3. Section 6.5

2.

$$f_{ave} = \frac{1}{4-1}\int_1^4 \frac{1}{x}dx = \frac{1}{3}(\ln x)_1^4 = \frac{1}{3}\ln 4.$$

4.

$$\begin{aligned} g_{ave} &= \frac{1}{2-0}\int_0^2 x^2\sqrt{1+x^3}dx \\ &= \frac{1}{2}\int_1^9 \sqrt{u}\cdot\frac{1}{3}du \qquad (u=1+x^3, du=3x^2dx) \\ &= \frac{1}{6}(\frac{2}{3}u^{3/2})|_1^9 = \frac{1}{9}(27-1) = \frac{26}{9}. \end{aligned}$$