1. **Section 11.8**

4. \( \lim_{n \to \infty} \left| \frac{(-1)^{n+1} x^{n+1}/(n+2)}{(-1)^n x^n/(n+1)} \right| = \lim_{n \to \infty} \left| x \right| \cdot \frac{n+1}{n+2} = \left| x \right| \). By the Ratio Test, the radius of convergence is 1. When \( x = 1 \), the series converges by the Alternating Series Test; when \( x = -1 \), the series diverges because it is the harmonic series. Thus the interval of convergence is \((-1, 1]\).

6. \( \lim_{n \to \infty} \left| \frac{\sqrt[n]{x^n+1}}{\sqrt[n]{x^n}} \right| = \lim_{n \to \infty} \left| x \right| \cdot \sqrt[n]{1 + \frac{1}{n}} = \left| x \right| \). By the Ratio Test, the radius of convergence is 1. When \( x = \pm 1 \), \( \lim_{n \to \infty} \sqrt[n]{x^n} = \infty \), so the series diverge by the Test for Divergence. Thus the interval of convergence is \((-1, 1]\).

8. \( \lim_{n \to \infty} \left| \frac{n^n x^n}{x^n+1} \right| = \lim_{n \to \infty} \left| x \right| \cdot \frac{1}{n} = \infty \). By the Ratio Test, the radius of convergence is 0, and the interval of convergence is \(\{0\}\).

14. \( \lim_{n \to \infty} \left| \frac{(2x^{n+1}/(2n+1))}{(2x^n/(2n))} \right| = \lim_{n \to \infty} \left| \frac{x^{2n+1}}{(2n+1)(2n+2)} \right| = 0 \). By the Ratio Test, the radius of convergence is \(\infty\) and the interval of convergence is \((-\infty, \infty)\).

16. \( \lim_{n \to \infty} \left| \frac{(n+1)^3(x-5)^n n^n}{n^3(x-5)^n} \right| = \lim_{n \to \infty} \left| x - 5 \right| \cdot \left(1 + \frac{1}{n}\right)^3 = \left| x - 5 \right| \). By the Ratio Test, the radius of convergence is 1. When \( x = 6 \) or 4, \( \lim_{n \to \infty} n^3(x-5)^n = \infty \), so the series diverge by the Test for Divergence. Thus the interval of convergence is \((4, 6]\).

24. \( \lim_{n \to \infty} \left| \frac{(2n+1)(2n+2)}{2^n} x^{n+1} \right| = \lim_{n \to \infty} \left| x \right| \cdot \frac{n+1}{2n^2} = 0 \). By the Ratio Test, the radius of convergence is \(\infty\) and the interval of convergence is \((-\infty, \infty)\).

2. **Section 11.9**

4. Since \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) exactly for all \( x \) with \( |x| < 1 \) and the power series converges exactly on \((-1, 1]\), we have \( f(x) = \frac{3}{1-x^2} = 3 \sum_{n=0}^{\infty} (x^2)^n = \sum_{n=0}^{\infty} 3x^{2n} \) for all \( x \) such that \( |x^2| < 1 \) \( \iff |x| < 1 \) and the interval of convergence for this power series is \((-1, 1]\).
8. Since \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) exactly for all \( x \) with \( |x| < 1 \) and the power series converges exactly on \((-1, 1)\), we have \( f(x) = \frac{x}{4x+1} = x \sum_{n=0}^{\infty} (-4)^n x^{n+1} \) for all \( x \) such that \(|-4x| < 1 \iff |x| < \frac{1}{4} \) and the interval of convergence for this power series is \((-\frac{1}{4}, \frac{1}{4})\).

24. Since \( \ln (1-t) = -\sum_{n=1}^{\infty} \frac{t^n}{n} \) for all \( t \) with \(|t| < 1 \) and the radius of convergence for the power series is 1, we have \( \frac{\ln(1-t)}{t} = -\sum_{n=0}^{\infty} \frac{t^n}{n+1} \) for all \( t \) with \(|t| < 1 \) and the radius of convergence for this power series is also 1. Thus, we have \( \int \frac{\ln(1-t)}{t} \, dt = C - \sum_{n=0}^{\infty} \frac{t^{n+1}}{(n+1)^2} = C - \sum_{n=1}^{\infty} \frac{t^n}{n^2} \) for all \( t \) with \(|t| < 1 \) and \( C \) being a constant and the radius of convergence for this power series is also 1.

3. Section 11.10

4. \( f^{(2n)}(x) = (-1)^n 2^{2n} \sin 2x \) and \( f^{(2n+1)}(x) = (-1)^n 2^{2n+1} \cos 2x \) for all nonnegative integers \( n \). Thus \( f^{(2n)}(0) = 0 \) and \( f^{(2n+1)}(0) = (-1)^n 2^{2n+1} \).

So the Maclaurin series of \( f(x) \) is \( \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{(2n+1)!} x^{2n+1} \). Since
\[
\lim_{n \to \infty} \left| \frac{(-1)^{n+1} 2^{2n+3} x^{2n+3}/(2n+3)!}{(-1)^n 2^{2n+1} x^{2n+1}/(2n+1)!} \right| = \lim_{n \to \infty} \frac{4x^2}{(2n+2)(2n+3)} = 0,
\]
the radius of convergence for this power series is \( \infty \) be the Ratio Test.

6. \( f^{(n)}(x) = (-1)^{n-1} (n-1)! (1+x)^{-n} \) for all positive integers \( n \). Thus \( f^{(n)}(0) = (-1)^{n-1} (n-1)! \) for all positive integers \( n \). So the Maclaurin series of \( f(x) \) is \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n \). Since
\[
\lim_{n \to \infty} \left| \frac{(-1)^n x^{n+1}/(n+1)}{(-1)^{n-1} x^n/n} \right| = \lim_{n \to \infty} \frac{n|x|}{n+1} = |x|,
\]
the radius of convergence for this power series is 1 be the Ratio Test.

8. \( f^{(n)}(x) = (x+n)e^x \) for all nonnegative integers \( n \). Thus \( f^{(n)}(0) = n \). So the Maclaurin series of \( f(x) \) is \( \sum_{n=0}^{\infty} \frac{n}{n!} x^n = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} x^n \). Since
\[
\lim_{n \to \infty} \left| \frac{x^{n+1}/n!}{x^n/(n-1)!} \right| = \lim_{n \to \infty} \frac{|x|}{n} = 0,
\]
the radius of convergence for this power series is \( \infty \) be the Ratio Test.

12. \( f(x) = x^3 = ((x+1) - 1)^3 = -1 + 3(x+1) - 3(x+1)^2 + (x+1)^3 \) for all \( x \). Thus \( -1 + 3(x+1) - 3(x+1)^2 + (x+1)^3 \) must be the Taylor series of \( f \) centered at \(-1\).
16. $f^{(2n)}(x) = (-1)^n \sin x$ and $f^{(2n+1)}(x) = (-1)^n \cos x$ for all nonnegative integers $n$. Thus $f^{(2n)}\left(\frac{\pi}{2}\right) = (-1)^n$ and $f^{(2n+1)}\left(\frac{\pi}{2}\right) = 0$. So the Taylor series of $f(x)$ centered at $\frac{\pi}{2}$ is

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(x - \frac{\pi}{2}\right)^{2n}.$$ 

40. Since $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all $x$, we have $\frac{\sin x}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$ for all $x$. Thus, we have

$$\int \frac{\sin x}{x} \, dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!} \cdot \frac{x^{2n+1}}{2n+1}$$

for all $x$ and $C$ being a constant.

56. Since $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ for all $x$, we have $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{6^{2n}(2n)!} = \cos \frac{x}{6}$. 

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}. $$