SOLUTION FOR HOMEWORK #10

1. Section 11.5

6. $\frac{1}{3n-1}$ is decreasing and $\lim_{n\to\infty} \frac{1}{3n-1} = 0$. Thus the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n-1}$ is convergent.

10. $\lim_{n\to\infty} \frac{\sqrt{n}}{1+2\sqrt{n}} = \lim_{n\to\infty} \frac{1}{2+1/\sqrt{n}} = \frac{1}{2} \neq 0$. Thus the series $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$ diverges.

2. Section 11.6

2. $\lim_{n\to\infty} \frac{(n+1)^2/2^{n+1}}{n^2/2^n} = \lim_{n\to\infty} \frac{1}{2}\cdot(1+\frac{1}{n})^2 = \frac{1}{2} < 1$. Thus the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges absolutely by the Ratio Test.

4. $\lim_{n\to\infty} \frac{2^n}{n^4} = \infty$. Thus $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^n}{n^4}$ diverges.

8. $\lim_{n\to\infty} \frac{n/(n^2+1)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$. Since the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by the limit comparison test. Since $(\frac{x}{x^2+1})' = \frac{1-x^2}{(x^2+1)^2} \leq 0$ for $x \geq 1$, $\frac{n}{n^2+1}$ is decreasing. As $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$, the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$ converges by the alternating series test. Thus $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$ converges conditionally.

20. $\lim_{n\to\infty} \sqrt[n]{\frac{1}{(\ln n)^n}} = \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1$, so the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\ln n)^n}$ converges absolutely by the Root Test.

3. Section 11.7

2. $\lim_{n\to\infty} \frac{(n-1)/(n^2+n)}{1/n} = \lim_{n\to\infty} \frac{n^2-n}{n^2+n} = 1$. Since the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the series $\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$ diverges by the limit comparison test.

6. $\lim_{n\to\infty} \sqrt[n]{(\frac{3n}{1+8n})^n} = \lim_{\infty} \frac{3n}{1+8n} = \frac{3}{8} < 1$, so the series $\sum_{n=1}^{\infty}(\frac{3n}{1+8n})^n$ converges absolutely by the Root Test.

8. $\lim_{k\to\infty} \frac{(2^{k+1}(k+1)!)/((k+3)!)}{(2^k k!)/((k+2)!)} = \lim_{k\to\infty} \frac{2(k+1)}{k+3} = 2 > 1$, so the series $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$ diverges by the Ratio Test.

12. Since $(\frac{x}{x^2+25})' = \frac{25-x^2}{(x^2+25)^2} \leq 0$ for $x \geq 5$, $\frac{n}{n^2+25}$ is decreasing from $n = 5$ on. As $\lim_{n\to\infty} \frac{n}{n^2+25} = 0$, the series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+25}$ converges by the alternating series test.

32. $\lim_{n\to\infty} \sqrt[n]{\frac{(2n)^n}{n^{2n}}} = \lim_{n\to\infty} \frac{2n}{n^2} = 0 < 1$, so the series $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ converges by the Root Test.