Math 620 Midterm Exam #2- March 30, 2012

1. (20 points) Suppose $f(x) \in K[x]$ is irreducible in $K[x]$ and L is a field extension of K of finite degree relatively prime to the degree of $f(x)$. Prove that $f(x)$ remains irreducible in $L[x]$.

2. (20 points) Suppose the degree $[F(\alpha) : F]$ is odd. Prove that $F(\alpha) = F(\alpha^2)$.

3. (20 points) Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 + x^2 + 1$.

4. (30 points) Prove that $\mathbb{Q}(\sqrt{2+\sqrt{2}})$ is a Galois extension of $\mathbb{Q}$ and compute the Galois group. Illustrate the Galois correspondence.

5. (10 points) Suppose $[K : F] = 2$ have characteristic zero. Prove that K is a Galois extension of F.