Math 620 HW1- Due Monday January 30

1. Page 918 # 2.

2. Let $\mathcal{C}$ be a category and $\{A_i \mid i \in I\}$ a family of objects of $\mathcal{C}$. A **product** for the family is an object P of $\mathcal{C}$ (usually denoted $\Pi_{i\in I}A_i$) together with a family of morphisms $\{\pi_i : P \to A_i \mid i \in I\}$ such that for any object B and family of morphisms $\{\phi_i : B \to A_i \mid i \in I\}$, there is a unique morphism $\phi : B \to P$ such that $\pi_i \circ \phi = \phi_i$ for all $i \in I$.

a. Describe a product for $\{A_1, A_2\}$ in terms of commutative diagrams.

b. Show that in the category of groups, $G_1 \times G_2$ with the usual projections maps π_1, π_2 is a product for $\{G_1, G_2\}$.

c. Come up with a definition of **coproduct** by reversing arrows in the definition of product. You can look it up if you are not sure.

d. Show that $Z_2 \times Z_3$ is a coproduct for Z_2 and Z_3 in the category **Ab** but not in the category of finite groups.

3. Let Ab be the category of abelian groups. For $n > 0$ consider the functor

$$F_n \colon \mathrm{Ab} \to \mathrm{Ab}$$

such that

$$F_n(G) = \{g \in G \mid ng = 0\}$$

Determine the left adjoint of F_n and prove that it is the left adjoint.

4. Give an example of a functor which is not representable, and prove that it is not.

5. Prove that the forgetful functor from groups to sets is representable.

6. Let T be the forgetful functor form the category of abelian groups to the category of groups. Let S be the abelianization functor $S(G) = G/G'$. Prove that (S, T) is an adjoint pair.

7. Define a contravariant functor $\mathcal{F} : \mathrm{Grp} \to \mathrm{Sets}$ as follows. Let $\mathcal{F}(G)$ be the set of subgroups of G. For $\phi : G \to H$ let $\mathcal{F}(\phi) = \phi^{-1}$. Is $\mathcal{F}$ representable?