Math 620 - Due Tuesday January 27

1a. (Hint: everything in this problem is totally straightforward!) Let $\mathcal{F} : \mathcal{C} \to \mathcal{D}$ be a functor and let $X \in ob\mathcal{D}$. In class we defined a universal arrow from X to $\mathcal{F}$. Now suppose $\mathcal{G}$ is a functor from $\mathcal{D}$ to $\mathcal{C}$ and let $A \in ob\mathcal{C}$. Define a universal arrow from $\mathcal{G}$ to A by reversing arrows.

b. Let $\mathcal{C}$ and $\mathcal{D}$ be categories. Describe the *product category* $\mathcal{C} \times \mathcal{D}$ in the obvious way. Let $\Delta : \mathcal{D} \to \mathcal{D} \times \mathcal{D}$ be the diagonal functor taking A to (A, A) and $f : A \to B$ to $(f, f) : (A, A) \to (B, B)$.

c. Let $A_1, A_2 \in ob\mathcal{D}$. Show that $(V, v), v = (v_1, v_2)$ is a universal arrow from Δ to (A_1, A_2) if and only if (V, π_1, π_2) is a product of A_1 and A_2 in $\mathcal{D}$.

2. Suppose $\mathcal{F} : \mathcal{C} \to \mathcal{D}$ is left adjoint to $\mathcal{G} : \mathcal{D} \to C$. Show that $\mathcal{G}$ preserves products. That is, suppose $\{A_i \in \mathcal{D}\}$ and $A = \Pi_{i \in I} A_i$ with maps $\pi_i : A \to A_i$ is a product in $\mathcal{D}$. Prove that $\mathcal{G}(A) = \Pi_{i \in I} \mathcal{G}(A_i)$ with maps $\mathcal{G}(\pi_i) : \mathcal{G}(A) \to \mathcal{G}(A_i)$ is a product in $\mathcal{C}$. Dualize to show $\mathcal{G}$ preserves coproducts.

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