Math 620 - Due Tuesday January 27

1a. (Hint: everything in this problem is totally straightforward!) Let \( F : \mathcal{C} \to \mathcal{D} \) be a functor and let \( X \in \text{ob}\mathcal{D} \). In class we defined a universal arrow from \( X \) to \( F \). Now suppose \( G \) is a functor from \( \mathcal{D} \) to \( \mathcal{C} \) and let \( A \in \text{ob}\mathcal{C} \). Define a universal arrow from \( G \) to \( A \) by reversing arrows.

b. Let \( \mathcal{C} \) and \( \mathcal{D} \) be categories. Describe the **product category** \( \mathcal{C} \times \mathcal{D} \) in the obvious way. Let \( \Delta : \mathcal{D} \to \mathcal{D} \times \mathcal{D} \) be the diagonal functor taking \( A \to (A, A) \) and \( f : A \to B \) to \( (f, f) : (A, A) \to (B, B) \).

c. Let \( A_1, A_2 \in \text{ob}\mathcal{D} \). Show that \((V, v), v = (v_1, v_2)\) is a universal arrow from \( \Delta \) to \( (A_1, A_2) \) if and only if \((V, \pi_1, \pi_2)\) is a product of \( A_1 \) and \( A_2 \) in \( \mathcal{D} \).

2. Suppose \( F : \mathcal{C} \to \mathcal{D} \) is left adjoint to \( G : \mathcal{D} \to \mathcal{C} \). Show that \( G \) preserves products. That is, suppose \( \{A_i \in \mathcal{D}\} \) and \( A = \Pi_{i \in I} A_i \) with maps \( \pi_i : A \to A_i \) is a product in \( \mathcal{D} \). Prove that \( G(A) = \Pi_{i \in I} G(A_i) \) with maps \( G(\pi_i) : G(A) \to G(A_i) \) is a product in \( \mathcal{C} \). Dualize to show \( G \) preserves coproducts.