# Math 619 Midterm Exam #1- September 27, 2012

**1. Short Answer- no work need be shown. (30 points)**

a. Define what it means for a group action to be *faithful.*

b. Determine the commutator subgroup of $D_8$ and $S_4$.

c. State the second (aka diamond) isomorphism theorem.

d. Give the conjugacy classes of $D_8$.

e. Define the center of a group.

**2. (20 points)** For $g \in G$ define

$$i_g : G \to G$$

by $i_g(x) = gxg^{-1}$. Recall that $\mathrm{Aut}(G)$ is the group of automorphisms of $G$.

a. Prove $i_g$ is an automorphism of $G$.
b. Prove the map $i : G \to \mathrm{Aut}(G)$ given by $g \to i_g$ is a group homomorphism.
c. What is the kernel of $i$?
d. Prove that the image of $i$ is a normal subgroup in Aut(G). This subgroup is called the group of *inner automorphisms.*

**3. (15 points)**Let $F \cong \mathbb{Z}/2\mathbb{Z}$ be the field with 2 elements. Write down the six elements in the general linear group $GL_2(F)$ and prove this group is isomorphic to $S_3$ by giving an explicit isomorphism.

**4. (15 points)** Give the entire subgroup lattice for a cyclic group of order 12.

**5. (10 points)** Let $f : G \to H$ be a homomorphism of groups, and let $B \leq H$. Prove that $\ker f$ is a normal subgroup of $G$ and $f^{-1}(B)$ is a subgroup of $G$.

**6. (10 points)** Give a presentation of the alternating group $A_4$ using only two generators.